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**“ASSESSMENT OF STABILITY OF PANEL IN AN
UNDERGROUND COAL MINE USING NUMERICAL
MODELLING”**

PROJECT PHASE-2 REPORT

(17MNP78)

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in partial fulfilment of the requirement for the award of the degree of

Bachelor of Engineering in

Mining Engineering

Under the Guidance of

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2020-21**



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CERTIFICATE

It is certified that the Project work entitled "**ASSESSMENT OF STABILITY OF PANEL IN AN UNDERGROUND COAL MINE USING NUMERICAL MODELLING**" is a bonafide work carried out by AURANGAZIB A (USN-1GV17MI005), SHIVA P N (USN-1GV17MI015), PRAVEEN KUMAR Y (USN-1GV17MI016) and NAVINPRASATH G S (USN-1GV15MI024) in the fulfilment for the award of the degree of Bachelor of Engineering in Mining Engineering of the Visvesvaraya Technological University, Belgaum during the year 2020-2021. It is certified that all corrections/suggestions indicated for the assessment have been incorporated in the report deposited in the departmental library. The project has been approved as it satisfies the academic requirement in respect of Project Phase-2-17MNP85 prescribed for the Bachelor of Engineering Degree.

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ABSTRACT

The success rate of underground coal mining at a greater depth of coverage depends on the safe and risk-free mining environment. The stability of a panel is an uncontrollable event in deep-seated underground mining. Hence, the evaluation of the stability of a panel of an underground coal mine helps to reduce risk and improve the safety of the workplace. To estimate the panel stability of an underground mine, the numerical modelling technique is a promising tool for conducting a parametric study to understand the behaviour of the working place for distinctive mining conditions. The physic-mechanical properties like depth of the seam, rock mass rating, Young's modulus of the coal seam, floor and roof plays a major role in the stability of a panel in an underground mine. Hence, a comprehensive study to assess the panel stability of a bord and pillar mining method is done to understand the behaviour of the rock mass for increasing safety and ensuring the production for varying geo-mining conditions of underground coal deposits by using numerical modelling.

Keywords:

Panel Stability, Numerical Modelling, Underground Coal Mining, Local Mine Stiffness, Strain Energy

1. INTRODUCTION

Mining may well have been the second of humankind's earliest advancement granted that agriculture was first. For the development of primary and basic industries of early civilization, mining and agriculture ranked together. From prehistoric times to the present, mining has played a major part in human existence (11).

The abundance of minerals and also energy provides a method of creating wealth. Minerals can be marketed on the open market, enabling the countries that possess them to obtain the valuable currency of the countries. The ability to use minerals and energy resources as a means of creating wealth opens the possibility that a given country and creating an economic cartel in that mineral.

By India's archaeological relics evidence, they strongly suggest that in the remote period of Country's history the utilization of coal has been increased. The coal mining industry emerged as a basic industry and by the second half of the 19th-century coal mining has spread in different countries world and in range of 1835 to 1882 the volume of world coal production went up from 36 million tons to 422 million tons and coal became a primary source of energy for the industrialized countries of the world. In 1900, about 95% of the world's commercial energy was derived from coal, only 4% from oil and gas, and less than 1% from hydraulic sources (9).

The mining of coal in India was started in the year 1774 on a small scale in the Raniganj coalfield. In 1900 coal production rose to 6.096 million tonnes. The increase in production was closely linked to the development of railways in India. Mining was confined to shallow depths and the policy of "more hole-more coal" was followed. Bord and pillar method was the method of mining, though all longwall face was reported at Narsimuda colliery. The whole part is developed into pillars without barriers. A typical layout of workings in the olden days. Coal was cut manually and loaded in tubes until the beginning of the present century mechanization was not exist (23).

Although bord and pillar mining methods predominant methods for underground mining, experimental longwall face mining is started at the seam depth. During the 2nd world war, open-pit mining was also introduced. In some mines mechanization with

coal cutting machines, conveyors, electric dill machines introduced for coal loading gathering arm loaders, or Huwood loaders were used. The growth of the coal mining industry and technological development were accelerated after India got independent in 1947.

The coal can be extracted by determining the many geological factors. The bord and pillar and longwall are the primary methods to mine the coal, In coal mining, the pillars play a major role in various purposes e.g., protection of mine shafts, panel isolation to guard against the spontaneous heating, proception of gallery and roadways, to manage the surface subsidence. The structural integrity of a coal mine largely depends on the pillars, the area of pillar strength and design is research has been done over the few decades. This project tells about the safe underground working when the coal seam is extracted by the bord and pillar method. It is necessary that evaluate the safety of the pillar through numerical modelling and can be approximate to that of a mine. The method used for calculating the FOS of the rib pillar for safety analysis. Usually, the percentage of extraction is around 60% in Indian mines by using Bord and Pillar method. This thesis also deals with pillar strength by different researches or academicians etc. and the determination of FOS.

At present, coal has been shared as a source of primary energy 30% and global electricity generation 41%. By the year 2030, the coal is forecasted to rise over 50%, the developing countries for 97% of the increase in coal production to improve the electrification rates (WCA, November 2010). As compared to the world wide Indian has the fifth-largest coal reserves, as of March 2017, 609.2 metric tonnes (25).

1.1 Background

Mining is one of the most important sectors for the progressive development and growth of the nation. Mining can be done in two ways i.e. underground and open cast. Underground coal mining broadly consists of two types, I.e: the Longwall method and Bord& Pillar method. In thatBord& pillar mining is most common in India.

The Bord and Pillar method of mining is suited to work flat coal seams of average thickness and at shallow depths. The secret of successful Bord and Pillar mining is

selecting the optimum pillar size. If the pillar size is too small the mine will collapse. If the size of the pillar is too large valuable minerals are left behind then it reduces the production of mine.

In the bord and pillar (B&P) method, pillars are formed by driving galleries as per CMR-1957 (CMR, 2017). Most of the bord and pillar method of coal mining has been done in-depth less than 300 m as at greater depths pillars experience crush. However, in India in some cases, the depth of 600 m the Bord and pillar method is carried out by besetting with the problems of strata control. Sometimes, the low strength of coal limits the depth to which bord and pillar mining can be done. Also, seam highly prone to spontaneous heating should not be worked by bord and pillar methods. pillars known as barrier pillars to separate the panels. The barrier pillars are significantly larger than the “panel” pillars are sized to allow them to support a significant part of the panel and prevent the progressive collapse of the mine in the event of failure of the panel pillars (26).

To support the overburden in underground mine the coal pillars are normally left. Their factor of safety plays a major role in estimating the safety of working in underground mines. It leads to an increase in coal mining even at higher depths because of increased demand for coal in the industrial sector.

As compared to the opencast mining method and longwall mining method the Bord and Pillar mining method is very cheap, but in terms of safety of the method as comparing by opencast and longwall mining methods the bord and pillar are less safe to do mining. The depth of cover and roof rock quality these geological parameters are responsible for the roof fall. To prevent roof fall parameters to be considered like panel width, gallery width, barrier pillar, and gate roads are most important. Pillar design also plays an important role whereby its stability mainly depends on depth and roof quality. Though mining has advanced leaps and bounds yet mining activities remain hazardous. Extraction of mineral wealth from underground sources is filled with many uncertainties. Underground coal mining is one such example. Still, a substantial part of the coal is left to support the roof. A large amount of coal resources is left unmined. This leads to the loss of the country’s natural resources. For maximum extraction of coal, new methods should be introduced without compromising the

safety of miners. Inappropriate pillar extraction creates dynamic loading over both natural and applied supports. This dynamic loading during the fall endangers safety and production [22].

Laboratory modelling and field testing are attempted for better caving of different types of overlying strata during the final extraction of the coal seam. All these attempts are being made for a controlled interaction of underground mining with the local geology and rock mass [22]. We can improve the de-pillaring method through case studies and numerical modelling for diverse geo-mining conditions.

1.2 Problem Statement

The factor of safety of a pillar is the ratio between the strength of the pillar and the applied stress. When the applied stress exceeds the strength of the pillar, the pillar is subjected to fail, and the load that it carried will be transferred to neighbouring pillars. The additional load on these pillars may lead to their failure. This mechanism of pillar failure, load transfer, and continuing pillar failure can lead to the rapid collapse of very large areas of a mine [19].

In some cases, only a few tens of pillars might fail; however, in extreme cases, hundreds, even thousands, of pillars can fail. This kind of failure has many names—progressive pillar failure, massive pillar collapse, cascading pillar failure, or pillar run. Flying debris can seriously injure or kill mining personnel. CPF might also fracture large volumes of rock in the pillars and the immediate roof and floor, leading to the sudden release of large quantities of methane into the mine atmosphere and possibly a methane explosion [28].

Unscientific extraction leads to disasters, as happened in Colebrook coal mine in South Africa. It was on January 21st 1960, approximately 900 pillars caved in almost 180 meters underground. There were 1000 workers in that shift, in that 437 workers died at the site. Later, from the investigation, they found that the accident was due to cascading pillar failure. Where a few pillars fail initially and this increases the load on the adjacent pillar causing them to fail. This causes pillar collapse over an area covering 324 hectares. Factors contributing to the collapse included the process of ‘top coaling’, which raised the height of extraction and reducing the size of pillars [28].

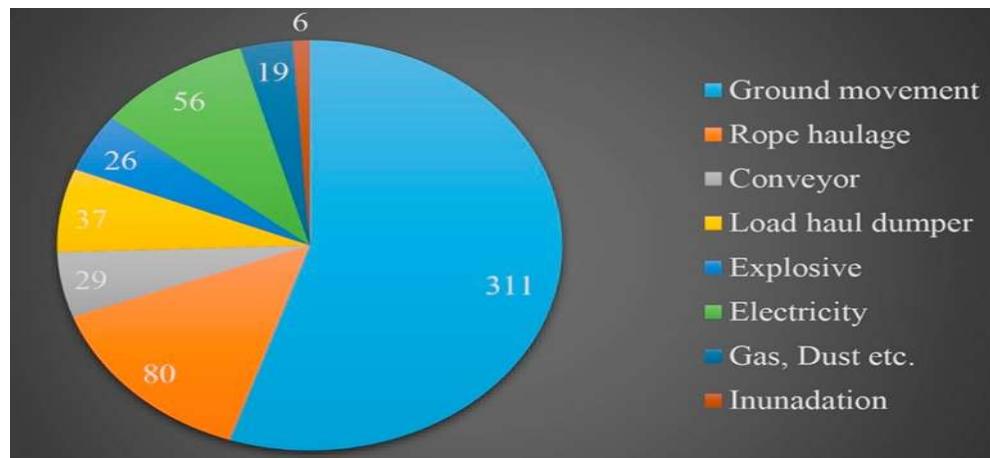


Figure.1.1: Cause wise analysis of fatal accidents in coal mines from 2001- 2014 [13].

Data of about 7000 accident reports of all the coal mining companies of India are collected from 2001-2014, which is shown in Figure.1.1. From the data, we can find that more than 50% of the accidents are due to ground movements [13].

1.3 Objective

- To access the stability of a panel in an underground mine.
- To determine the local mine stiffness and post-failure stiffness of a panel based on varying geo-mining conditions.
- To determine the strain energy accumulated in the panel based on varying geo-mining conditions.

1.4 Methodology

To achieve the above-said objectives, the following methodology was adopted.

In this study as a pre-requisite for this research work, an exquisite literature review is to be done considering all the past research work carried out by researchers, academicians, scientists etc. related to the present topic was reviewed covering underground coal mining, its stability and the different contributing factors affecting factor of safety are thoroughly studied through literature review. Geotechnical data is collected through a literature review to evaluate panel stability. Evaluation of the hazards of panel pillar failure is studied through numerical modelling for identified geo-mining conditions by considering the different geo-mining parameters. Based on the numerical modelling results, the prediction of possible panel design for a given

geo-mining condition by mathematical modelling will be developed as shown in Figure. 1.2.

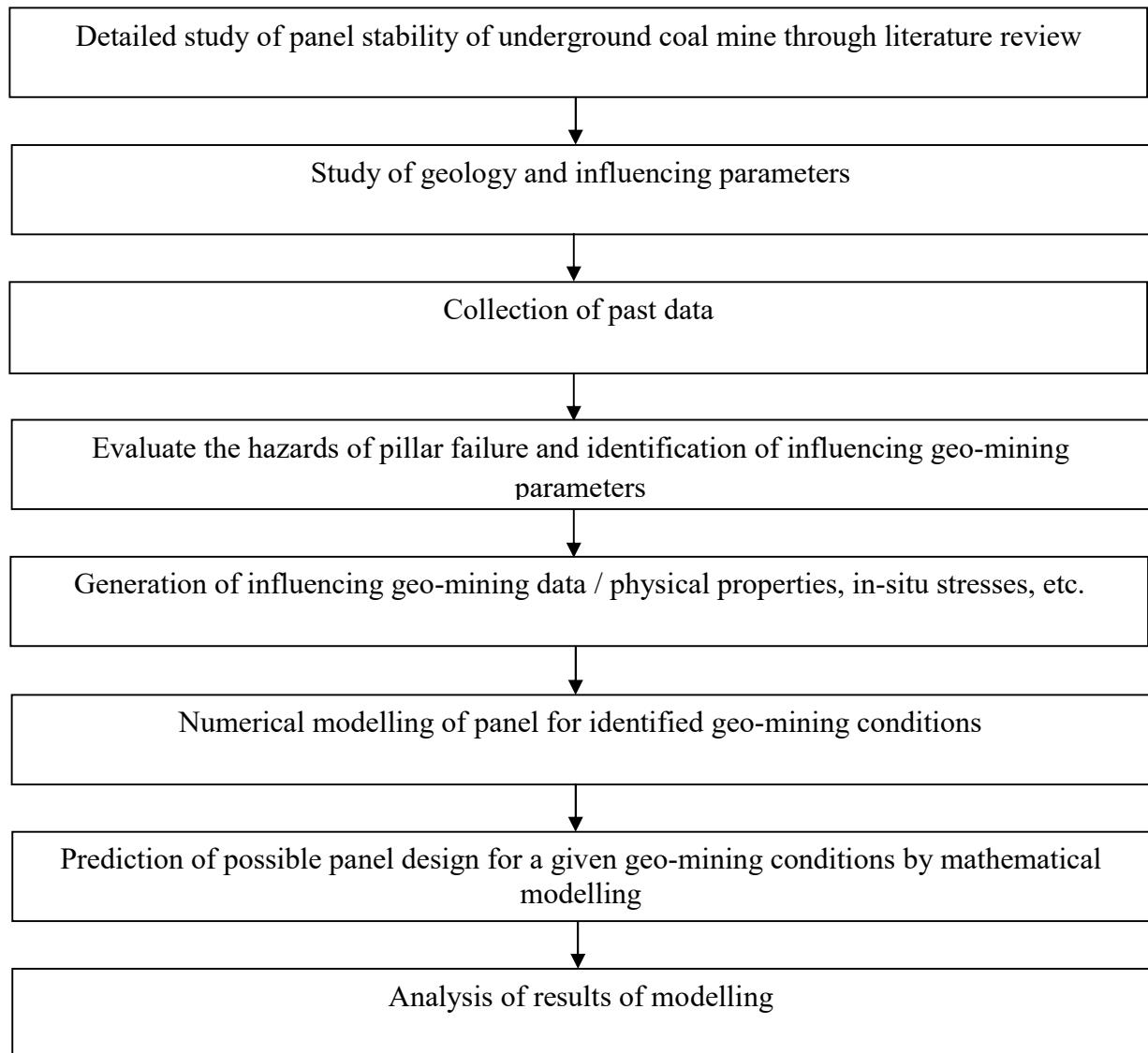


Figure. 1.2: Flow chart of the methodology adopted in the study

1.5 Scope of the Study

To assess the rock mass behaviour during the extraction of coal from underground coal mining, this project aims to analyse panel stability for different geo-mining conditions. The scope of this study is limited to analyze the stability of a panel with physicomechanical properties associated with an underground coal mine within the

domain of the properties selected through numerical modelling exercise. It is worth here mentioning that it is difficult to do a parametric study through numerical modelling of underground bord and pillar de-pillaring due to the requirement of very fast computing machines, huge memory and time. Hence, the scope of the research to be made in the proposal is bord and pillar.

1.6 Importance of the Study

The future of coal mining lies in the production of study/benefit to the study. Coal from deep mines for upcoming decades since reserves amenable for underground mining under the shallow depth of cover range are fast exhausting worldwide. One of the major stability to control problems associated with underground coal mine pillars, which should be scientifically addressed for better stability management and safer extraction of pillars. An in-depth R&D study on coal mine pillars is required to identify the problems and they're controlling also suitable for Indian geo-mining conditions. This study will help to control pillar failures and pillar instability in underground coal mines. It will increase the safety of the miners and mine workings. The productivity will be on the safer side with an advent increase in pillar failures control in underground coal mines.

1.7 Organization of the Study

The thesis consists of five chapters. In the first chapter, statements of the problem, the objective of the project, approach, scope of the work, the significance of the study are discussed. Chapter 2 presents the literature review on pillar design, pillar failure mechanism, factors influencing panel stability and numerical modelling using FLAC^{3D}. Chapter 3 reflects the Parametric study followed with step by step approach. Chapter 4 is a validation of the study. Results obtained from the models ran in FLAC 3D software were analyzed in this chapter. Chapter 5 presents the conclusion, in this chapter, the conclusion figured out from the results and analysis was displayed, and according to the results recommendations were suggested at last.

2. LITERATURE REVIEW

Pillar design is one of the basic elements of mining engineering. During underground mining, pillars support a large weight of overlying strata. Without stable pillars, ground control is impossible. During the 1990s lots of mines sized their pillars using local rules of thumb that were based on experience. Pillar recovery operations had been associated with about 25% of all roof fall fatalities underground [13].

2.1 Bord and Pillar Method

The development of mine by the method of working known as Bord and Pillar consists of driving a series of narrow roads, separated by blocks of solid coal, parallel to one another, and connecting them by another set of narrow parallel roadways driven nearly at right angles to the first set as shown in figure 1. The stage of formation of a network of roadways is known as the development of first working and these roadways are called Bord or Gallery. When the gallery is developed a solid block of coal is left surrounded by the gallery are known as Pillar. The coal pillars formed are extracted after the development of the mine leasehold and this later stage of extracting coal from pillars are known as depillaring. This method is sometimes called room and pillar mining. A normal layout of the bord and pillar method is given in Figure 2.1.

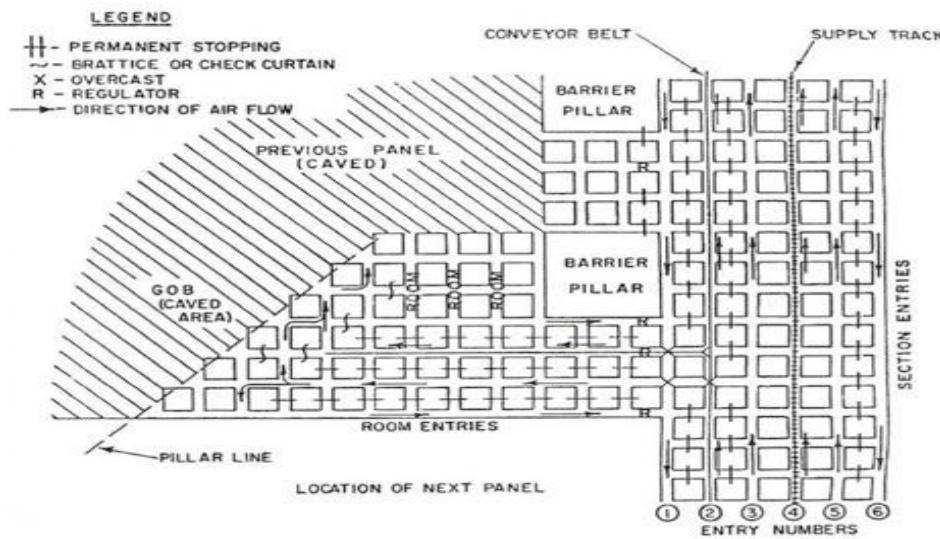


Figure 2.1: Schematic diagram of a typical bord and pillar working

2.2 Classification of Bord and Pillar Mining System

The Bord and pillar mining system can be done in three following ways:

1. Develop the entire area into pillars and then extract the pillars starting from the boundary.
2. Develop the area into panels and extract pillars subsequently panel-wise. This is called a panel system of mining.
3. The working of “Whole” followed by “Broken” working in which the mine is opened out by a few headings only and thereafter development and depilating go on simultaneously starting from the boundary(23).

2.3 Applicability of Bord and Pillar Method

The bord and pillar mining is adopted for working coal seams thicker than 1.5 meters, for seams free from stone or dirt bands and for flat seams of average and at shallow depth i.e., (<300 meters). The seams at moderate depth or higher depths i.e., (>600 meters) are developed by the bord and pillar method of mining and extracted by the blasting gallery method. Bord and pillar mining is applicable only for non-gassy coal seams with a strong roof and floor which can stand for a longer period after the development stage. In this method of mining, the coal should be of adequate crushing strength to prevent the premature collapse of the pillars or overlying strata.

2.4 Design of Bord and Pillar Workings

Design of bord and pillar working depends mainly on:

- The size of the panel
- The size of the barriers
- The size of the pillars and
- The width and height of galleries

2.4.1 Size of the panel

The main consideration in deciding the size of the panel is the incubation period of the coal seam. Since the size is so fixed that the entire panel can be extracted within the incubation period without the occurrence of spontaneous fire. The incubation of Indian coalfields generally varies from 6 - 12 months. The size of the panel could be increased depending upon the rate of extraction, i.e., using mechanized methods of extraction. Strata conditions also determine the panel dimensions.

2.4.2 Size of the barrier

The width of the barrier depends on the load which it has to carry and its strength. Greater the depth of working, wider the barrier, and also softer the coal, more is the width of the barrier. In practice, the width of the barrier enclosing pillars in a panel is usually the same as is the width of the coal pillars which are enclosed within the panels.

2.4.3 Size of pillars

The size of the pillars are influenced by the following:

- i. Depth from the surface and percentage extraction in the first workings or development.
- ii. Strength of coal: Seams with weak coal require large pillars. The effect of the atmosphere and the escape of gas also influence the size of the pillars.
- iii. The nature of the roof and floor. These influence the liability to crush and creep. A strong roof tends to crush the pillar edges whilst a soft floor predisposes it to creep and both call for larger pillars.
- iv. Geological considerations: In the vicinity of faults, larger pillars are required. Dip and the presence of water also influence the decision as to the size of the pillars.
- v. Time-dependent strain: While time strain goes on increasing, the load remaining constant and if the size of the pillar is not sufficiently large, then it may fail under the time-dependent strain, although initially, it might be stable.

With the passage of time, spalling and weathering takes place which reduces the strength of coal pillars.

2.4.4 Width and height of galleries

The width and height of the galleries must be specified and in this case, the other parameters such as the size of panels, size of pillars, and barriers also determine the width and height to be considered for galleries or roadways in the bord and pillar method of mining. Indian Coal Mine Regulations 1957, restricts the width of galleries to a maximum of 4.8 meters(23).

2.5 Statutory Guidelines

In India, the dimensions of pillars and the width and height of galleries are regulated by the Govt of India i.e. DGMS vide its Regulation 99 of Coal Mines Regulation1957(Table 2.1and 2.2). The width of galleries should not exceed 4.8m and the height of the galleries should not exceed 3m. For the width of galleries ranging from 3 m to 4.8 m, the dimensions of pillars for various depths of working are given below.

Table 2.1: Pillar distance (centre to centre) concerning the depth

Depth of the seam from the surface	Where the width of the galleries do not exceed			
	3 m	3.6 m	4.2 m	4.8 m
	The distance between the centres of adjacent pillars shall not be less than (in m)			
Not exceeding 60 m	12	15	18	19.5
Between 60 -90 m	13.5	16.5	19.5	21
Between 90- 150 m	16.5	19.5	22.5	25.5
Between 150- 240 m	22.5	25.5	30.5	34.5
Between 240 -360 m	28.4	34	39.5	45
Exceeding 360 m	39	42	45	45

Table 2.2: Percentage of extraction for different depths are tabulated below

Depth of the seam from the Surface	Where the width of the galleries do not exceed			
	3 m	3.6 m	4.2 m	4.8 m
	Percentage of Extraction (%)			
Not exceeding 60 m	43.7	42.2	41.2	43.17
Between 60 -90 m	39.53	39.8	38.4	40.5
Between 90- 150 m	33.06	33.5	33.8	34
Between 150- 240 m	24.8	26.2	25.6	25.9
Between 240 -360 m	9.95	19.7	20.1	20.2
Exceeding 360 m	14.8	16.4	17.8	19

2.6 Basic Principles of the Pillar Design

Pillar is insitu rock remnants left between adjacent underground openings. Rock pillar mainly serves as an underground supporting element in large underground space. Without a pillar, it is very difficult to ground the weight of overburdened material or withstands lateral pressure in the deep underground opening.

Pillar design is an essential exercise for mining engineers for estimating the factor of safety of underground working during development and depilating. The pillar is designed to be carried out for both underground coal and metal mines. The conventional theory proposes that local stability is ensured if pillar strength exceeds the stress place on it.

The ratio of pillar's estimated strength to the pillars stress is expressed as the factor, pillar strength and pillar load have to be known. The process of designing pillars involves determining their proper size according to the accepted load on the pillars.

2.6.1Pillar design

The pillar load might be estimated from tributary area theory, also the pillar strength from empirical formulas and laboratory coal strength testing. In recent times, powerful design approaches have been developed after the analysis of large databases of real-

world pillar successes and failures. These contain the Analysis of Retreat Mining Stability (ARMPS), the Analysis of Longwall Pillar Stability (ALPS), the Mark - Bieniawski rectangular pillar strength formula, and guidelines for avoiding massive pillar collapses. A different model divides pillars into three categories:

- **Slender pillars** ($w/h < 3.0$), when these pillars are loaded to their maximum capacity, they fail, shedding nearly their entire load.
- **Squat pillars** ($w/h > 10$), these pillars can carry very large loads, and may even be strain-hardening (meaning that they may never actually shed load, but just may become more deformable once they “fail”)
- **Intermediate**, are those whose w/h ratios fall between about 4 and 8, these pillars do not shed their entire load when they fail, but neither can they accept any more load.

Several pillar design formulas were suggested in the early period, based upon laboratory testing, full-scale pillar testing, and back-analysis of mine case histories (12). They were technologically advanced for an industry that depends almost completely on Bord and pillar mining at comparatively shallow depth. The energy disaster of the 1970s and 1980s faced the renewal of attention in coal pillar design. Several aspiring field studies were carried out, numerous of them sponsored or led by the U.S. Bureau of Mines. By 1980, the classic pillar design methodology had completely developed.

It comprised of three stages:

- Estimating the pillar load by tributary area theory
- Estimating the pillar strength with a pillar strength formula
- Computing the pillar safety factor (SF).

Various formulas were accessible for the estimation of pillar strength as a function of two variables, the pillar’s width to height ratio (w/h) and the coal seam strength calculated from laboratory testing (Bieniawski, 1984), Arthur Wilson of the British National Coal Board was the first to yield a completely different approach to pillar design. His analytic method preserved the pillar as a complex structure, with a non-identical stress gradient, a build-up of confinement about a high-stress core, and

progressive pillar failure. Even if his mathematics were seriously weak(20), Wilson's basic concepts are now mostly accepted and inspire nearly all modern numerical models. Since 1990, the number of pillar strength formulas and numerical models had increased, but their forecast for squat pillars varied broadly. One study associated 10 formulas, and establishes that some expected that pillar strength would increase exponentially as the w/h ratio increased, it would incline towards a maximum limiting value, and still, others expected a midway, linear increase.

2.6.2 Physical Properties of Rock Material

The physical properties of rocks affecting design and construction in rocks are;
Composition

- Structure
- Texture
- Mineralogical
- Specific gravity (G)
- Unit weight
- Density
- Void ratio (e)
- Porosity (n)
- Moisture content (w)
- Degree of saturation, (S)
- Coefficient of Permeability (k)
- Electrical and Thermal properties
- Swelling
- Anisotropy
- Durability
- **The mineralogical composition** is the intrinsic property controlling the strength of the rock Although there exist more than 2000 kinds of known minerals, only about nine of them partake decisively informing the composition of rocks
- **Specific gravity** is the ratio of the density of solids to the density of water.

- **Density** is a measure of mass per unit of volume. The density of rock material varies, and is often related to the porosity of the rock. It is sometimes defined by unit weight and specific gravity. Most rocks have density between 2,500nd 2,800 kg/m³.
- **Void ratio(e)** is the ratio of the volume of voids (VV) to the volume of solids (VS)
- **Porosity (n)** describes how densely the material is packed. It is the ratio of the non-solid volume (VV) to the total volume (V) of material. Porosity, therefore, is a fraction between 0 and 1
- **Porosity** decreases with increasing age of the rock and depth of the rock
Porosity is a measure of water – holding capacity of a rock material
- **Moisture Content (M):** it is the ratio of the weight of water in the voids to the weight of dry solids in the rock sample
- **Degree of saturation (S):** it is defined as the volume of water in the void to the total volume of voids in the rock sample, the rock mass having higher porosity has a higher degree of saturation
- **Permeability (k):** the ability of porous material to allow a liquid to pass through its pores, units: cm/sec, or m/sec.

$$Q = k i A$$

Q= discharge through the area, i= hydraulic gradient

- **Electrical properties:** Most of the rocks are dielectric in nature and measurement of Dielectric constants is used for data interpretation Electric resistivity method used in geophysical prospecting
- **Thermal Properties:** Increase in temperature makes rock weaker due to the formation of cracks in the rock mass
- **Coefficient** of thermal expansion of the rocks: increase in length due to a change in temperature
- **Swelling:** it is an increase in the volume of the mass due to suction of water or due to contact of water for a long time, Swelling is more in weaker type rocks
- **Anisotropy:** properties of the elements of the rock mass are not similar in every direction, due to the sequence of rock formation, i.e., due to the existence

of bedding planes, etc. Anisotropic material has some weakness in a particular direction. Sedimentary rocks have a high degree of anisotropy.

- **Durability:** it is the resistance to destruction. If the rock is more durable means it will last for a longer period when put into use. It depends upon the nature of the environment against which the rock is going to be used. Swelling index or slake durability test is used to describe the nature of weathering.

2.6.3 Mechanical Properties of Rock Material

The mechanical or strength properties of rocks are;

Strength: Ability of a material to resist an externally applied load, but In Rock mechanics, strength is the Force per unit Area required to bring about rupture in a rock mass at a given environmental condition.

Classification of strength: depending upon the type of loading and the stresses, the strength, in general, may be classified as

- Compressive Strength
- Tensile strength, and
- Shear Strength

For determining the above strength values the tests are conducted either on intact rock specimens in the laboratory tests or on rock mass in the field, i.e., in situ strength tests

In the laboratory, there are direct Methods for the determination of above strength values and also indirect methods for the determination of above strength values roughly in the laboratory or at the field site.

Compressive strength

The compressive strength of a material is a measure of its ability to resist uniaxial compressive loads without yielding or fracture. The most common measure of compressive strength is the Uniaxial compressive strength or unconfined compressive strength. It is one of the most important properties used in design, analysis, and modelling.

Direct Methods:

- a) Uni axial Compression Test
- b) Triaxial Compression Test

Indirect Method:

- a) Point Load Test
- b) Schmidt hammer Test

Direct Method: It requires the preparation of samples by ISRM (International Society of Rock Mechanics).

- a) **Uniaxial compressive strength (UCS)** of rock material and deformation behaviour under loading is verified by applying compressive load until failure occurs in the core by a fracture in the middle using high capacity Compressive testing machines.
- b) **Triaxial Compression strength (TCS)**

When the rock specimen is subjected to confining pressure in addition to vertical pressure, the strength exhibited by the rock specimen is known as Triaxial compressive strength.

Indirect method

a) Point Load Strength Index Test

The point load test of rock cores can be conducted diametrically and axially. In the diametrical test, the rock core specimen of diameter D is loaded between the point load apparatus across its diameter. The length/diameter ratio for the diametrical test should be greater than 1.0.

b) Schmidt or rebound Hammer Test

It normally tests on the surface hardness of the rock sample as it is also easy to use and handle. The sample can be in core or block shape and it is a non-destructive type of test. The best part of the test is that the sample used for the previous test can be used again.

Tensile strength test

Tensile strength of a material is defined as the maximum tensile stress which a material is capable of developing in the natural rock mass is rarely subjected to direct tension, but it is subjected to tensile stresses Rocks are weak in tension

Shear strength Test

Shear strength may be defined as the maximum resistance to deformation due to shear displacement caused by shear stress. Shear strength in a rock mass is derived from the surface frictional resistance along the sliding plane, interlocking between individual rock grains and cohesion in the sliding surface of the rock.

2.7 Rock Mass Classifications

Rock masses are complex systems with high variation between them, making them difficult to describe quantitatively. Because each rock mass can be described uniquely, a means of classifying rock masses is necessary to categorize and group them. This classification, in addition to easing and standardizing communication regarding rock masses, is valuable during the engineering design process (7).

2.7.1 Rock Quality Designation (RQD)

One of the oldest rock mass classification systems which are still widely used today is the rock quality designation (RQD) system. The RQD system, which was developed as a technique for quantifying the percentage of the recoverable core, was first described by Don Deere in 1966. The RQD value, which is calculated by analyzing core samples, is equal to the ratio of the cumulative length of core greater than 100 mm (4 inches) to the total length of the core. Pieces greater than or equal to 4 inches in length are considered to be “sound” core, and smaller pieces are the result of shearing, jointing, faulting, or weathering within the rock mass. In addition to the numerical value of RQD, Deere has categorized ranges of values and suggested qualitative descriptions.

When RQD was introduced, there were many existing methods for estimating the core recovery percentage, but RQD became the standard and a widely used index of rock quality. The RQD index became a standard because it is easy to measure, easy to calculate, and non-destructive. Because of its applicability and simplicity, it has been incorporated as one input parameter into more involved rock classification systems (7).

Table 2.3: Qualitative descriptions of rock quality designation range as suggested by Deere

RQD %	Qualitative Description
0 - 25	Very Poor
25 - 50	Poor
50 - 75	Fair
75 - 90	Good
90 - 100	Excellent

2.7.2 Rock Mass Rating (RMR)

One such rock mass classification system which includes the RQD among many other parameters is the rock mass rating (RMR) system. The RMR system was originally called the “Geomechanics Classification” by Bieniawski, its developer [5].

Table 2.4: Qualitative descriptions associated with rock masses with RMR ranges.

Rating	Class no.	Qualitative Description
100 - 81	I	Very Good Rock
80 - 61	II	Good Rock
60 - 41	III	Fair Rock
40 - 21	IV	Poor Rock
< 20	V	Very Poor Rock

Table 2.5: Quantitative estimates of rock mass strength parameters, rock mass cohesion, and rock mass friction angle, as well as an expected life of a tunnel through a rock mass based on rock mass rating ranges (6).

Class no	Average stand-up time	Cohesion (kPa)	Friction Angle (°)
I	20 yr. for 15-m span	> 400	> 45
II	1 yr. for a 10-m span	300-400	35-45
III	1 wk. for a 5-m span	200-300	25-35
IV	10 hr for 2.5 -m span	100-200	15-25
V	30 min for 1-m span	<100	<15

The form of the rock mass rating system used today includes six inputs: uniaxial compressive strength, rock quality designation (RQD), discontinuity spacing, condition of discontinuities, presence of groundwater, and orientation of joints. In addition to the passionate support of RMR, Bieniawski gave a detailed explanation of how to determine its value here [6]. The RMR system typically takes values between 0 and 100, like the RQD system. But unlike the RQD system, negative values are possible with RMR. Bieniawski groups rock masses within ranges of RMR values and assign qualitative descriptions, as well as some reasonable quantitative values, as shown in Tables 2.4 and 2.5.

The RMR system was originally developed as an aide for tunnel design and support selection. While the RMR system has not changed in its essential nature or purpose since its introduction, some modifications were made to it in its first ten to fifteen years in existence. Since its development, the RMR system has been adapted for use in many rock mass design applications including foundations and slopes as well as underground excavations other than simple tunnelling [4].

2.7.3 Mining Rock Mass Rating(MRMR)

This classification system was developed for mining purpose

In 1990 Laubscher, developed the Mining Rock Mass Rating (MRMR) system by modifying the Rock Mass Rating (RMR) system of Bieniawski. In the MRMR system the stability and support are determined with the following equations:

$$\text{RMR} = \text{IRS} + \text{RQD} + \text{spacing} + \text{condition}$$

In which,

RMR = Laubscher's Rock Mass Rating

IRS = Intact Rock Strength

RQD = Rock Quality Designation

spacing = expression for the spacing of discontinuities condition = condition of discontinuities (parameter also dependent on groundwater presence, pressure, or quantity of groundwater inflow in the underground excavation)

$$\text{MRMR} = \text{RMR} * \text{adjustment factors}$$

In which,

adjustment factors = factors to compensate for: the method of excavation, orientation of discontinuities and excavation, induced stresses, and future weathering

The parameters to calculate the RMR value are similar to those used in the RMR system of Bieniawski. This may be confusing, as some of the parameters in the MRMR system are modified, such as the condition parameter that includes groundwater presence and pressure in the MRMR system whereas groundwater is a separate parameter in the RMR system of Bieniawski. The number of classes for the parameters and the detail of the description of the parameters are also more extensive than in the RMR system of Bieniawski.

The basic functions of the MR-MR classification system are too

- subdivide (classify) the rock mass into zones, based on similar behaviour
- provide a basis for communication between various mining disciplines and
- formulate design parameters for the actual mine design.

The MRMR system is one of the methods to characterize rock mass competency. It is important to understand that rock mass competency is not only influenced by its inherent geological parameters (material strength and quantity and strength of the defects). but also, the change introduced by the mining activities (induced stress, blasting damage, exposure to weathering. relative orientation of the defects and excavations. water). These 'man-made' changes often have detrimental effects on the rock mass competency and thus the stability of the openings. and cannot be ignored.

The most common errors in classifying rock masses include

avenging values across geotechnical domains:

- Mixing natural and mining-induced defects (joints and fractures).
- Mixing in situ rock mass ruing (IRMR) and modified rock mass rating (MRMR) values.
- Not considering the variability (distribution) of values of individual parameters.
- Ignoring rock strength anisotropy and its orientation.
- Avenging the intact rock strength (IRS) of weak and strong zones.
- Averaging joint conditions for individual discontinuity sets.
- Not considering discontinuities other than joints.
- Ignoring the orientation of the structural irregularities (small and/or large-scale joint expressions).
- Ignoring or misusing the sampling error adjustment.
- Wrongly adjusting for alteration.
- Wrongly adjusting for weathering.
- Not recognizing internal rock defects such as discontinuous natural fractures. stuns. foliation. cemented joints. schistocyte. bedding. preferred mineral orientation and microfractures.
- Applying mining adjustments without considering the spatial relationship and time leg weathering, blasting).
- Mixing localized failures with casing.
- Altering the classification system to suit the local 'needs' and then using a stability graph, and ground support tables based on original (unaltered) ratings.

2.8 Extraction of Pillars

After the formation of the pillar, their extraction is done from one end of the panel. If the development was not done in panels, artificial panels of suitable sizes are created by building stopping around the pillars intended to be extracted such that the extraction of all the pillars of the panel is completed within the incubation period under regulation 137 of the Coal Mining Regulation 2017. Further Regulation 112 of CMR 2017 lays down certain conditions which must be complied with during the extraction. Some of the statutory requirements are given below.

- No extraction or reduction of pillars shall be commenced, conducted or carried out except with the previous permission in writing of the Regional Inspector and by such conditions as he may specify therein.
- An application, for permission under sub-regulation (1) shall be accompanied by two copies of an up-to-date plan of the area where pillars are proposed to be reduced or extracted, showing the proposed extension of extraction reduction of pillars, how such extraction or reduction is to be carried out, the thickness and depth of the seam, the nature of the roof, and the rate and direction of dip.
- The extraction of a reduction of pillar shall be conducted in such a way as to prevent, as far as possible, the extension of collapse or subsidence of the goaf over pillars that have not been extracted.
- Save as otherwise provided under sub-regulation(5), no pillars shall be reduced or split in such a manner as to reduce the dimensions of the resultant pillars below those required by regulation 111 or by any order made thereunder, nor shall any gallery be so heightened as to exceed three meters.
- During the extraction of pillars, no splitting or reduction of pillars or heightening of galleries shall be effected for the distance greater than the length of two pillars ahead of the pillars that are being extracted or reduced:
 - a) Provided that where pillar extraction is about being in a district, such splitting or reduction of pillars or the heightening of galleries shall be restricted to a maximum of four pillars.
- The width of the split-galleries shall not exceed the width specified for galleries under sub-regulation(4) of regulation 111 of CMR.

- The Regional inspector may, by an order in writing and stating the reasons therefore, relax or restrict the provisions of sub-regulation(5) in respect of any specific workings to such extent and on such conditions as he specifies therein.
- Where the method of extraction is to remove all the coal or as much of the coal as practicable and to allow the roof to cave in, the operations shall be conducted in such a way to leave as small an area of the un-collapsed roof as possible with due regard to danger from an air blast or weighting on pillars, suitable means shall be adopted to bring down the goaf at regular intervals wherever possible.
- Where the voids formed as a result of extraction are stowed with sand or other materials, the owner, agent or manager shall, or before the 10th day of every month submit to the regional inspector a statement giving the quantity of coal raised and the quantity of sand or other material stowed in every district during the preceding month.

2.9 Problems Faced During the Extraction of Pillars

The operations of pillar extraction are beset with the problems of strata control. If the operations have not been designed scientifically, there are the dangers of major strata movement set in, which may result in the overriding of pillars, and premature collapse. In the past and also recent years in the Jharia coalfields and elsewhere during the extraction of pillars in thick seams, especially seams developed in multi-sections, premature collapses have occurred involving large areas.

Maintenance of an acceptable environment is not easy, splitting of pillars provides many leakage routes heightening and widening of galleries increase cross-sectional areas, and hence the velocity of ventilation air is reduced, the ventilation in depillaring faces other becomes sluggish. Airborne dust concentration increases and climatic generally become uncomfortable.

Usually, some coal is left in the goaf, which may be 15-20% of the panel reserve, this gets crushed, oxidation sets in and eventually fire may break out, there are numerous causes of a fire occurring in depillaring districts in Indian coal mines. Mechanization of coal getting is not easily possible on account of the difficulty of roof control.

Compared with the other methods of coal extraction, it offers the advantages of great operational flexibility, relative freedom in the sequence of seam extraction, insensitivity to local and regional geological disturbances, maintenance of the integrity of the roof strata and surface, and, finally, low capital intensity. The main disadvantages of bord-and-pillar mining are that coal has to be left in situ to support the roof strata and that labour productivity is relatively low when compared with opencast and longwall mining systems. It is important to note that both the amount of coal being lost in the support pillars and the labour productivity are dependent on the depth of mining.

Different approaches for calculating pillar stability are done, two of the main approaches are –Salamon proposed a criterion for assessing mine stability using the concept of mine stiffness. Maleki proposed a strata stiffness Index, the ratio of strata stiffness-strength to that of the coal seam, for identifying bump-prone conditions. Using empirical formulations proposed by Bieniawski or Salamon, for instance, the confining stresses are not directly taken into account for estimating peak pillar strength by using site-specific for-field horizontal stress and material properties [19].

2.10 Pillar Failure

Pillar failure generally leads to loss of support which in turn causes roof fall. This creates fractures and other geological disturbances in the overburden. These fractures are the main cause of roof fall and consequent sinkholes. Moreover, the failure of one pillar transfers the load to surrounding pillars and may lead to progressive pillar failure (sudden or gradual) or excessive displacements over a relatively large area. Pillar failure occurs when the load on the pillar is more than the strength of the pillar. Crushing of pillars occurs due to an increase in existing loads, chemical oxidation of coal, mine fires, and flooding of mines. In addition to pillar strength, the pillar width to height ratio (w/h) is also important[12]. For —slender pillars ($w/h < 4$), failure often results in nearly complete loss of load-bearing capacity, sometimes with sudden and total collapse. Pillars with w/h between about 4 and 10 are largely elastic with a possible plastic core, and failures tend to occur gradually with post-failure residual strength essentially constant. The pillars deform until they have shed enough load to stop the process. Pillars with w/h greater than 10 have a plastic core and may strain harden once the loss of initial strength due to crushing or yielding of the outer elastic

portion of the pillar occurs. After this initial crushing, the pillars gain strength as they deform. The implications for surface structures of the failure of slender pillars with shallow cover are much more significant than those associated with the yielding of squat pillars at great depth. Different formulas for analyzing the strength of a pillar have been developed, and computer programs for performing pillar analyses are available. Pillar stability formulas can be divided into two categories – analytical and empirical. Analytical formulas involve extensive material testing, the understanding of loading under varying conditions, and a safety factor of around 2 based on knowledge and understanding of all variables. One of the first analytical models developed for estimating pillar strength is Wilson's approach, which can be directly calculated, hence making it more flexible and adaptable to actual conditions compared to any other empirical equation. It can be used to estimate the stress distribution from the edge of a pillar to the centre based on the confined core theory. Wilson's equation uses the Mohr-Coulomb failure criterion for modelling the coal and surrounding rock; however, at high confinement (high w/h ratio) coal strength is not linear with the result that it overestimates pillar strengths [12].

2.11 Pillar Failure Mechanism

As suggested by Tincelin and Sinou [24], pillar failures can be classified into two categories:

Slow, progressive deterioration of the pillars causes relatively delayed surface subsidence and even damage if the pillars fail. It is also called controlled pillar failures which occur gradually and typically over long periods. These pillar failures are also termed creep and squeeze. Sudden, violent collapse of pillar causing immediate surface damage and mostly associated with fatal accidents. This is called uncontrolled pillar failure and take place suddenly and violently and fall into the second group of pillar failures. Uncontrolled pillar failures occur rapidly and may not be preceded by any deterioration of the pillars[8].

- Stable, nonviolent failure occurs when $|KLMS| > |KP|$
- unstable, violent failure occurs when $|KLMS| < |KP|$

- Where $|KLMS|$ is local mine stiffness and $|KP|$ is post- failure stiffness at any point along the load-convergence curve of the pillar as shown in Fig 2.2.[28].

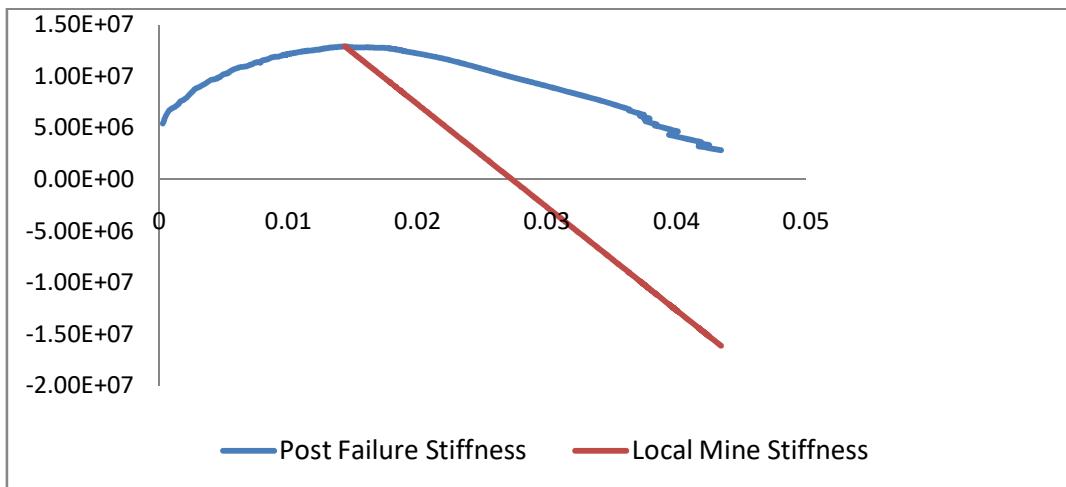


Figure.2.2: Schematic representation of stable nonviolent failure

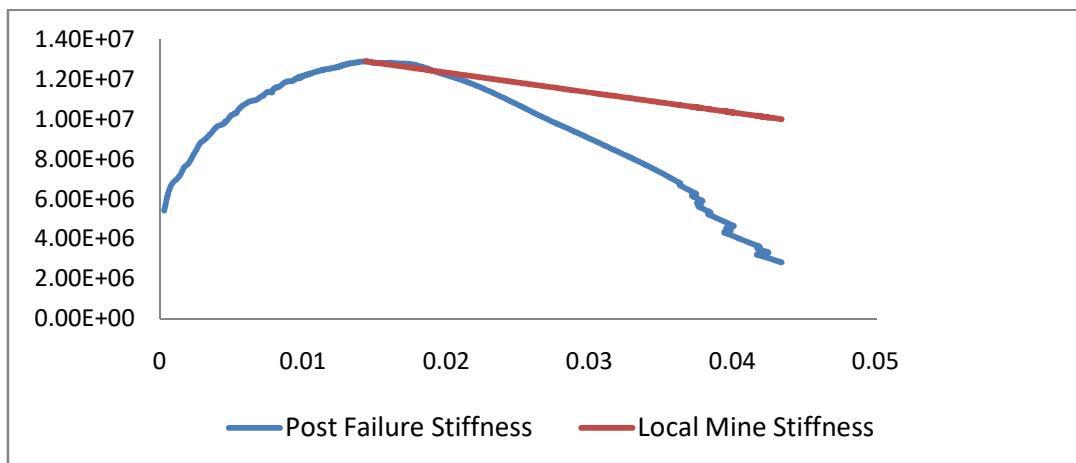


Figure.2.3: Schematic representation of unstable violent failure

The Controlled and uncontrolled pillar failures depend on two main factors: local mine stiffness and post-failure stiffness. The w/h ratio is the main controllable parameter that governs the post-failure stiffness of the pillars while the local mine stiffness may vary due to many reasons: mine layout, major geological structures, extraction ratio.

2.12 Pillar Analysis

The performance of a pillar is controlled by several factors, including the inherent strength of coal, fracturing, internal friction angle, cohesion, surrounding strata conditions, pillar geometry and roof/pillar/floor contact conditions. According to Salamon, while pillars with intermediate w/h ratios ($3 < w/h < 5-7$) can fail suddenly and violently as a whole, wide pillars ($w/h > 7$) would only suffer from side failures and pressure bursts but complete pillar failure would not occur [10].

The study by Hill on pillar failure says that,

1. Over 50% of the failed pillar cases have a design SF of <1.5 and a pillar w/h ratio <2 .
2. The density of failed cases starts to reduce for w/h ratios >2 and is effectively almost 0 for values >5 .
3. The only failed case at a w/h ratio of >5 (approximately 8) had an SF < 1 and was likely to be a floor-bearing failure rather than a core pillar failure[8].

2.13 Factors Influencing Panel Stability

1. Depth from the surface and percentage extraction in the first workings or development. As depth increases, the load on the pillar increases.
2. Strength of the coal: Seams with weak coal require large pillars. The effect of atmosphere and escape of gas also influence the size of pillars.
3. The nature of the roof and floor: These influence the liability to crush and creep. A strong roof tends to crush the pillar edges whilst a soft floor predisposes it to creep and both call for large pillars.
4. w/h ratio: if this ratio is more than 8, then the pillar is stable. If the ratio is less than 2, then the pillar is unstable.
5. Geological Considerations: In the vicinity of faults, large pillars are required. Dip and presence of water also influence the decision as to the size of the pillars.
6. Time dependant strain: With time the strain goes on increasing, the load remaining constant and if the size of the pillar is not sufficiently large, then it may fail under the time dependant strain, although initially, it might be stable [14].

2.14 Numerical Modelling

The approach of the numerical method is to divide the problem into small physical and mathematical components and then combine the all influence of the components to approximate the behaviour of the whole system. The series of full mathematical equations is formed in this process then solved approximately. Various numerical modelling technique has been developed and currently being used worldwide. The methods are categorized as a continuum, discontinued and the hybrid continuum or discontinued [10].

2.14.1 Basics of numerical modelling

The continuum postulation implies that all points in a problem region cannot be open or broken into pieces. All material points originally in the neighbourhood of a certain point in the problem region, remain in the same neighbourhood throughout the deformation. The continuum problem can be solved by three different methods:

- Finite Element Method(FEM)
- Finite Difference Method(FDM)
- Boundary Element Method(BEM)

Conventional methods for determining local mine stiffness and post-failure stiffness is expensive and time-consuming. By using Fast Lagrangian Analysis of Continua in 3 Dimensions (FLAC^{3D}), we can easily determine the local mine stiffness and post-failure stiffness by numerical modelling. Within this scope, numerical models enhanced with well-quantified geotechnical parameters can provide useful insights. These numerical models can also be utilized to assess other controlling parameters of pillar stability such as contact conditions, floor behaviour, the effect of horizontal discontinuities, water and/or gas content in the rock mass[18].

A computer simulation, a computer model or a computational model is a computer program or network of computers, that attempts to simulate an abstract model of a particular system. Models can take many forms, including but not limited to dynamical systems, statistical models, differential equations, or game-theoretic models. Often when engineers analyze a system to be controlled or optimized, they use a mathematical model. From the analysis, engineers can build a descriptive model of the

system as a hypothesis of how the system could work, or try to estimate how an unforeseeable event could affect the system. Similarly, in control of a system, engineers can try out different control approaches in simulations. A mathematical model usually describes a system by a set of variables and a set of equations that establish relationships between the variables. The values of the variables can be practically anything; real or integer numbers, boolean values or strings, for example. The variables represent some properties of the system, for example, the measured system outputs often in the form of signals, timing data, contours, and event occurrence. The actual model is the set of functions that describe the relations between the different variables. Here FLAC (FastLagrangian Analysis of Continua) has been used for simulation and analysis[10].

2.14.2 FLAC 5.0

FLAC is a two-dimensional explicit finite difference program for engineering mechanics computation. This program simulates the behaviour of structures built of soil, rock or other materials that may undergo plastic flow when their yield limits are reached. Materials are represented by elements, or zones, which form a grid that is adjusted by the user to fit the shape of the object to be modelled. Each element behaves according to a prescribed linear or nonlinear stress/strain law in response to the applied forces or boundary restraints. The material can yield and flow and the grid can deform (in large-strain mode) and move with the material that is represented. The explicit, Lagrangian calculation scheme and the mixed-discretization zoning - 33 - the technique used in FLAC ensure that plastic collapse and flow are modelled very accurately. Because no matrices are formed, large two-dimensional calculations can be made without excessive memory requirements. The drawbacks of the explicit formulation (i.e., small-timestep limitation and the question of required damping) are overcome to some extent by automatic inertia scaling and automatic damping that does not influence the mode of failure. Though FLAC was originally developed for geotechnical and mining engineers, the program offers a wide range of capabilities to solve complex problems in mechanics. Several built-in constitutive models that permit the simulation of highly nonlinear, irreversible responses representative of geologic, or similar, materials are available. However, it offers several advantages when applied to engineering problems[10].

3. PARAMETRIC STUDY

A Parametric study that perturbs design variables in the product design model to explore design alternatives can effectively support product concept designs. The advantage of a parametric study is that it allows us to nominate parameters for evaluation, define the parameter range, specify the design constraints, and analyze the results of each parameter variation[1].

This project was decided to go with the parametric study because of the advantage of the wide window it offers to understand and study different variable properties.

A parametric study can be done in different ways either the selected parameters of a study can be in a continuous variable, uniform interval variable, or ununiform interval variable. In this study, the uniform interval variable parametric study was adopted. The selected variable parameters were the properties of coal which will not be efficient to follow any other methods, other than that this method offers an advantage of plotting the intermediate values also with the help of the plotted graphs.

A parametric study with numerical modelling provides an efficient tool for this project. numerical modelling is known for its high efficiency in solving complex problems quickly.

3.1 Steps Involved in Numerical Modelling

These are the following steps used in Numerical Modelling:

1. Defining element types
2. Defining material parameters
3. Applying in Software
4. Creating the geometry model
6. Solution
7. Analysis of results

Numerical modelling has been used to predict/investigate bump proneness by estimating the amount of strain energy released, or by determining the local mine stiffness and comparing with the post-failure stiffness, or by large/rapid deformation of the roof, or based on stress-strain analysis, or energy release ratio[27]. Using the geo-mining conditions, numerical modelling can be used for scientifically predicting the impact of the identified parameters towards the post and pre-mining stresses on strata contributing to coal bumps using FLAC3D [10].

The prediction of bump proneness in numerical modelling involves three stages as follows:

Stage I – Using the numerical model to determine the local mine stiffness,

Stage II – Using the numerical model to determine post-peak failure stiffness and

Stage III – Comparing the tangent of local mine stiffness with post-peak failure characteristic curve

Based on the analogy between laboratory test specimens and mine pillars, Salamon (1970) developed a criterion i.e. if the local stiffness is lesser than the post-failure peak stiffness of the pillar as shown in Fig.3.1, then the pillar fails in an unstable manner or violently[28].

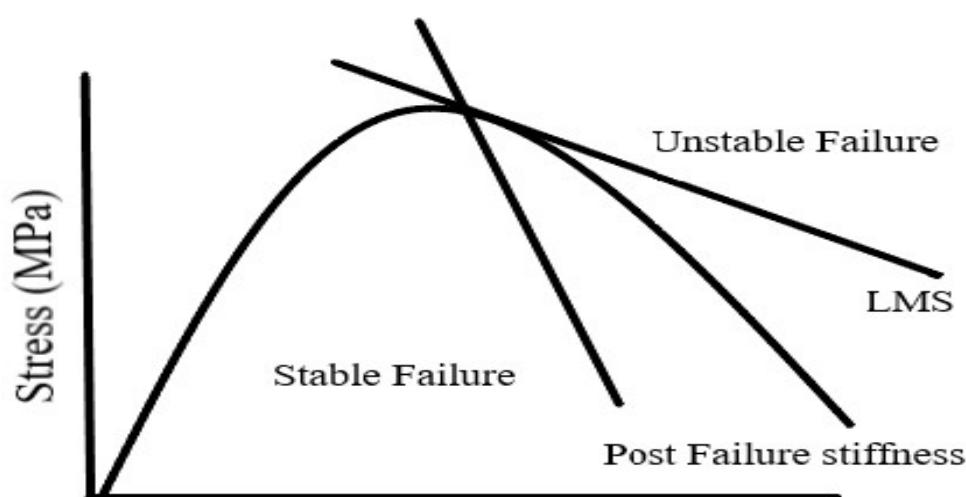


Figure.3.1: Stable failure or unstable failure depending upon the local mine stiffness and post-failure stiffness

Local mine stiffness can be described as the load-deformation characteristics between hanging and footwall or roof and floor. In assessing the stability of a mining structure like pillars and to analyse the coal bump, it is generally used to determine the local mine stiffness and to compare it with the post-failure stiffness of the pillar. If the local stiffness is lesser than the post-failure peak stiffness of the pillar, it results in violent failure [27].

Post-failure stiffness is the tangent of the sloping curve of the stress-strain curve of the pillar as shown in Fig.1. The post-failure stiffness may depend on the temperature, confining pressure and loading rate [28].

3.2 Selection of Suitable Material Model

A constitutive law is used to calculate the resulting change in stress caused by an increment in strain. Many different constitutive material behaviours may be applied within FLAC3D, however elastic, Mohr-Coulomb (plastic), and Mohr-coulomb strain-hardening/softening (brittle/weakening) models are used exclusively.

In mining, it was frequently observed that the induced loading or stress exceeds the strength of the rock mass. In this way, constitutive laws that can represent the reaction of the rock mass in the post-peak state are required for the practical representation of stresses and deformation in such circumstances[19].

There are two plasticity models available in FLAC^{3D} which are,

3.2.1 Mohr-Coulomb Model

The Mohr-Coulomb model is the conventional model used to represent shear yielding in soils and rocks.

In the implementation of the Mohr-Coulomb in FLAC3D, an elastic guess is first computed, by adding to the old stress components, increments calculated by application of Hooke's law to the total strain increment for the step. Principle stresses and corresponding principle directions are calculated and ordered. If these stresses violate the composite yield criterion, a correction must be applied to the elastic guess to give the new stress state.

The plastic strain is not calculated directly in this model, to speed the calculation. The strain-softening model can be used if the plastic is needed and/or gradual or no tensile softening is desired.

3.2.2 Mohr-Coulomb Strain- Hardening/ Softening (MCSS) Model

The stress-strain behaviour of unconsolidated rock mass is represented using Mohr-Coulomb constitutive more with a slight modification. The modification is based on the expansion/contraction of the yielding surface, which is based on the changes of the internal friction angle and the cohesion through plastic strain, i.e., mobilized strength properties. The modification is done to represent the hardening-strain and softening-strain behaviour of the rock mass [10].

The Mohr-Coulomb strain-softening (MCSS) model allows strength and flow properties to vary as functions of total plastic strains and provides tremendous control over the response of the constitutive model. The user enters tabular values for friction angle, cohesion, and dilation angle which are then used in the standard fashion within the Mohr-Coulomb constitutive law. This complete process is repeated at every time step through a central differencing scheme with the stress and strain calculations being performed at alternating half-time steps.

3.3 Determination of MCSS Parameters by the Single Pillar Test Run

Determining the Mohr-Coulomb strain-softening parameter in real practice is more difficult, which is done easily by the test run model. The number of test trials is carried out on the single pillar with different width and height ratios and suitable representative Mohr-Coulomb strain-softening parameters are estimated as by back analysis. The Mohr-Coulomb strain-softening properties like Cohesion, friction angle, and dilation angle is determined by a single pillar test run the model. This method of determining the MCSS parameters considered to be best practice [15]. A single pillar test run model is given in figure 3.2.

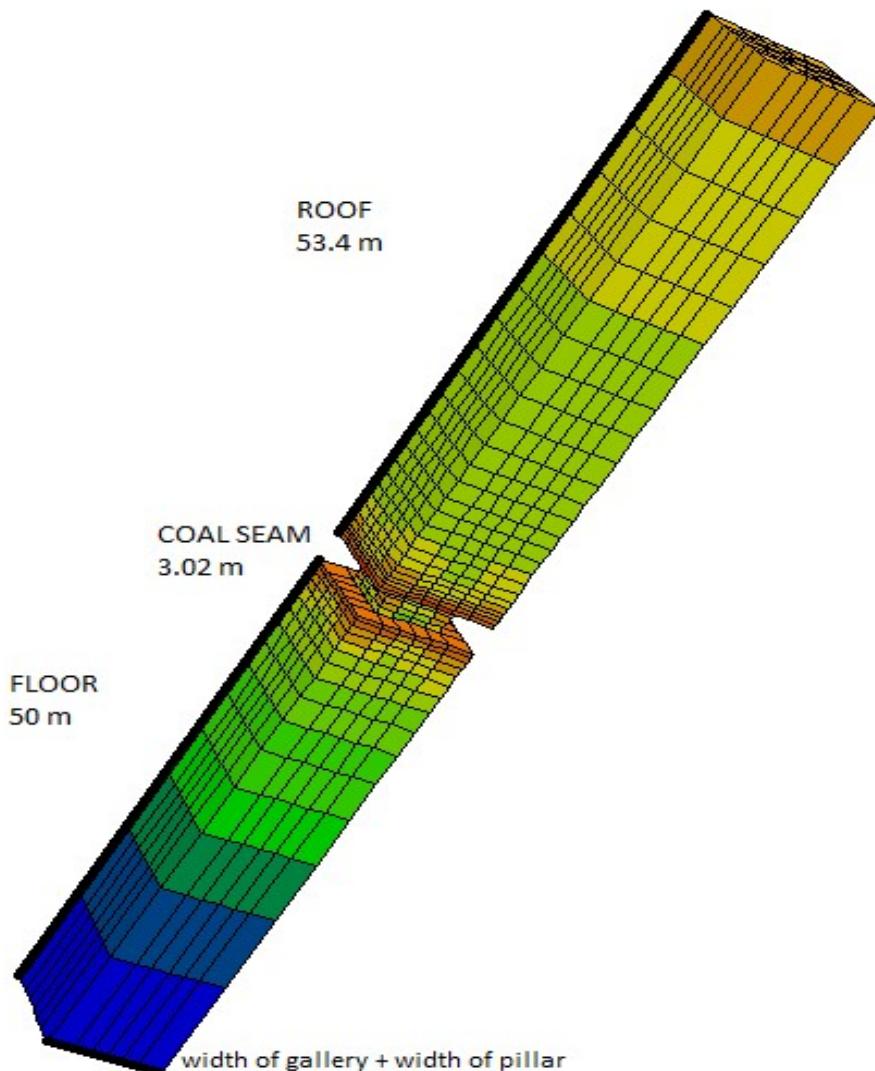


Figure 3.2: Numerical model for a single pillar

In-situ horizontal stress condition is an important parameter to design an underground mining structure. Based on a thermo-elastic shell model of the earth, Sheorey (1994) proposed an equation for the average in-seam horizontal stress. In this theory, it is observed that the mean in-situ horizontal stress (mean of the major and minor horizontal stresses) depends on the elastic constants (Young's modulus – E, Poisson's ratio – v), the coefficient of thermal expansion (β) and the geothermal gradient (G). This equation gives the value of mean horizontal stress as:

$$\sigma_h = \frac{v}{1-v} \sigma_v + \frac{\beta EG}{1-V} (H+1000) \text{ MPa}$$

where,

H = Depth of cover in meter,

σ_v = Vertical stress and

σ_h = Horizontal stress.

In the study by Sheorey et al. (2001), this equation is shown to fit stress measurement data from different parts of the world quite well. In absence of measured data for Indian coalfields, in-situ stresses are simulated. The vertical in-situ stress, induced due to gravity, is taken as:

$$\sigma_v = 0.025H \text{ MPa}$$

We obtain the mean horizontal stress as:

$$\sigma_h = 2.4 + 0.01H \text{ MPa}$$

Although the available numbers of in-situ stress measurement data for Indian coalfields are only a few, this equation has good agreement with some measured data in India. Among these, the measurements by erstwhile CMRI (CMRI Report, 2002) are of considerable importance and it is observed that the horizontal stress field is not highly anisotropic but supports. The shear strength and friction angle are estimated using Sheorey's failure criterion [21] for rock masses which follows the 1976 version of rock mass rating (RMR) of Bieniawski (1976) for reducing the laboratory strength parameters to give the corresponding rock mass values. This criterion is defined as:

$$\sigma_1 = \sigma_{cm} \left[1 + \frac{\sigma_3}{\sigma_{tm}} \right]^{bm}$$

where σ_1 and σ_3 are major and minor principal stresses at failure and the rock mass strength parameters are defined by:

$$\sigma_{cm} = \sigma_c \exp \left[\frac{RMR - 100}{20} \right]$$

$$\sigma_{tm} = \sigma_t \exp \left[\frac{RMR - 100}{27} \right]$$

$$bm = b^{RMR/100, bm < 0.95}$$

where,

σ_1 = Triaxial strength of rock mass, MPa

σ_3 = Confining stress, MPa

σ_c = Compressive strength of intact rock, MPa

σ_t = Tensile strength of intact rock, MPa

b = Exponent of intact rock which controls the curvature of the triaxial curve

σ_{cm} = Compressive strength of rock mass, MPa

σ_{tm} = Tensile strength of rock mass, MPa

RMR = Bieniawski (1976) rock mass rating

bm = Exponent for rock mass corresponding to the intact rock constant defined above.

In the above equations, the subscript m stands for the rock mass, where σ_c and σ_{cm} are the compressive strengths of intact rock and rock mass respectively. σ_t and σ_{tm} are tensile strengths of intact rock and rock mass respectively. σ_1 and σ_3 are major and minor principal stresses respectively at the time of failure b and bm are constants.

For estimating these parameters, only the value of the compressive strength is known. Then the $b = 0.5$ is taken as the most representative value, as seen from a large number of test data published earlier [21]. The rock mass shear strength τ_{sm} ; the coefficient, μ_{0m} and the angle of internal friction, ϕ_{0m} are obtained as:

$$\tau_{sm} = \left[\sigma_{cm} \sigma_{tm} \frac{bm^{bm}}{(1 + bm)^{1+bm}} \right]^{1/2}$$

$$\tau_{0m} = \frac{\tau_{sm}^2 (1 + bm)^2 - \sigma_m^2}{2\tau_{sm} \sigma_{tm} (1 + bm)}$$

$$\phi_{0m} = \tan^{-1}[\mu_{0m}]$$

It is observed that the values of shear strength τ_{sm} and friction angle ϕ_{0m} , so determined needs to be adjusted slightly. There is a slight adjustment required to incorporate the fact that the Mohr-Coulomb strain-softening plasticity model in FLAC3D uses the linear MohrCoulomb criterion, while the Sheorey criterion is non-linear [15].

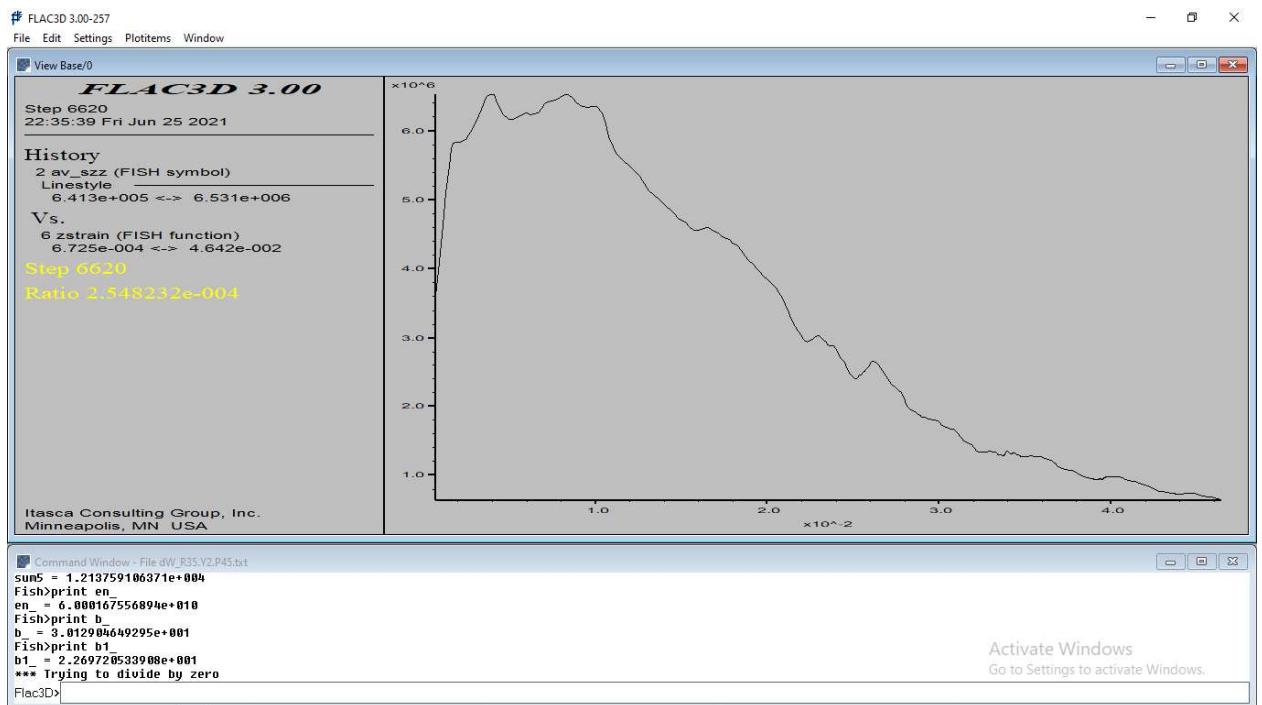


Figure 3.3: Average stress vs strain of a single pillar with w/h ratio of 2.

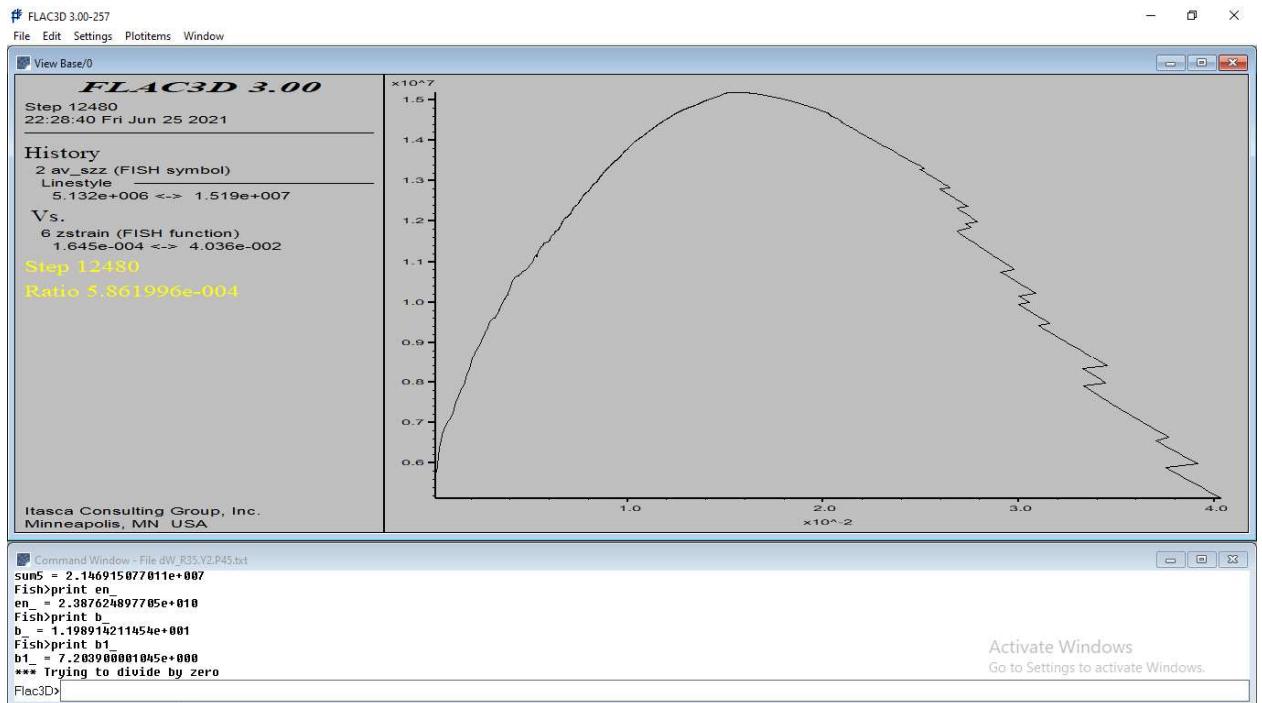


Figure 3.4: Average stress vs strain of a single pillar with w/h ratio of 5.

From the above figures 3.3 and 3.4, the analysis was done on a single pillar with varying width to height ratio, the maximum stress obtained in the pillar before failure was assumed to represent the strength of that pillar. This indicates that once the stress exceeded the strength, the pillar failed. The strength obtained from the pillar is then compared with strength obtained from formulas such as Sheorey's formula.

If the strength obtained in FLAC^{3D} does not match with the one calculated from formulas, then the modifications on cohesion, dilation, and frictional angle are done and the procedure is repeated for varying width to height ratios. An acceptable range of error is from -0.05 to +0.05. The values are shown in table 3.1.

Table 3.1: Comparison of strength using Sheorey's formula and using FLAC3D.

Compressive strength of coal (MPa)	Height of the pillar (m)	Depth of cover (m)	w/h	Width (m)	Strength of pillar using Sheorey's formula (MPa)	Stress obtained in FLAC3D (MPa)
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32.55	3.02	300	2	6.04	8.103537	8.095
32.55	3.02	300	2.5	7.55	9.203537	10.22
32.55	3.02	300	3	9.06	10.30354	11.83
32.55	3.02	300	3.5	10.57	11.40354	12.95
32.55	3.02	300	4	12.08	12.50354	13.69
32.55	3.02	300	4.5	13.59	13.60354	14.13
32.55	3.02	300	5	15.1	14.70354	14.42

The Rock Mass properties for different formations that are used in modelling are given in table 3.2.

Table 3.2: Rock Mass Properties Used for Different Formations

Formation	Thickness (m)	Young's Modulus (GPa)	Density (kg/m ³)	Uni-axial Compressive Strength (MPa)	Tensile Strength (MPa)	RMR
Floor	50	7.5	2578	95.70	8.84	51
Coal Seam	3.02	Variable from 2 to 4	1606	32.55	3.50	Variable from 35 to 65
Shale	0.8	3.98	2578	60.60	4.83	44
Fgsst	0.82	4.54	2278	95.70	8.84	42

Shale	1.21	4.0	2575	70.70	7.00	37
Cgsst	13.06	6.49	2158	38.40	3.88	55
Mgsst	39.90	6.99	2378	40.00	4.40	60

Table 3.3: Bulk and Shear Modulus Calculation Using Young's Modulus.

Formation	Young's Modulus (GPa)	Bulk's Modulus (GPa)	Shear Modulus (GPa)	Poison's Ratio
Floor	7.5	5.0	3.0	0.25
Coal seam	2	1.3	0.8	0.25
	3	2.0	1.2	
	4	2.6	1.6	
Shale	3.98	2.7	1.592	0.25
Fine grained sandstone	4.54	3.0	1.816	0.25
Shale	4.0	2.7	1.6	0.25
Coarse grained sandstone	6.49	4.3	2.596	0.25
Medium grained sandstone	6.99	4.7	2.796	0.25

3.4 Study for Panel Stability

Once the Mohr-Coulomb strain-softening parameter is determined, the same is used in the main model during the development of the gallery (4.8m). After the development

of the gallery, the load/average stress on the middle pillar is obtained. Using a FISH (an in-built programming language of FLAC3D) file the average vertical stress concentration on the pillar and the average convergence (roof and floor) is determined. Out of 15 pillars in the coal seam of the model, the middle pillar is removed as shown in Fig.8 to determine the average convergence between roof and floor.

The models give the roof-to-floor convergence C_p with the pillar in place, roof-to-floor convergence C_e with the middle pillar removed and σ_z the average vertical stress on the pillar. Then, the local mine stiffness is calculated from the equation given below.

$$K = \frac{\sigma_z \times A}{C_e - C_p}$$

Where

σ_z is the stress in the pillar before extraction

A is the plan area of the pillar.

C_e is the convergence of the central point of the pillar after extraction

C_p is the convergence of the central point of the pillar before extraction

3.5 Development of the Models

In this project, a panel of 15 pillars and 10 barrier pillars is considered for modelling. A panel of dimensions is considered. The models were created in FLAC 3D from the bottom to top the height of the model was 108.61m from -50 to 58.61 being the coal pillar at 0 to 3.02m covered with galleries on four sides after the development.

After forming the roadways in the model, the top of the model is fixed in the vertical direction and a constant velocity of 10.5 m/s is applied. Application of zero vertical displacements at the model bottom and zero horizontal displacements at the four vertical symmetry planes are the other boundary conditions adopted in the model as shown in Figure.3.5.

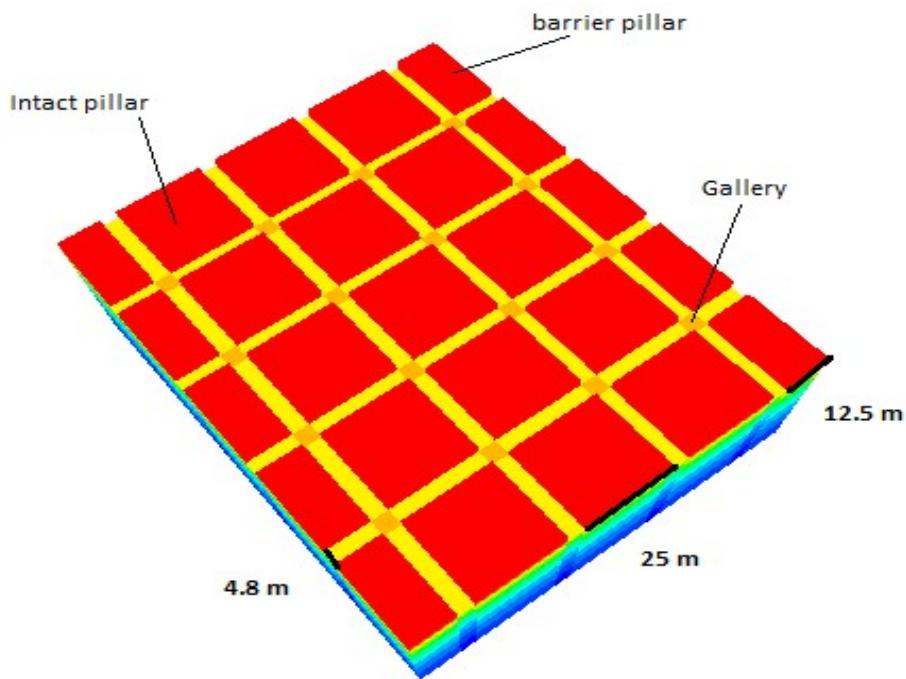


Figure 3.5: The numerical model using FLAC^{3D} for the developed coal seam

For the estimation of the pillar failure of a mine for a particular geo-mining condition, the stored strain energy before failure and the released strain energy after failure are determined from the stress-strain curve. If the stored strain energy is larger than the released strain energy then the mine may be considered as burst prone [20].

The central pillar is removed and allow the pillar to fail. The failure of the pillar is a gradual process, as observed during the model run, the pillar starts failing at the outer edges and proceeds towards the centre of the core, and the horizontal stress falls little due to the crushing of the pillar as shown in Figure 3.6 [16].

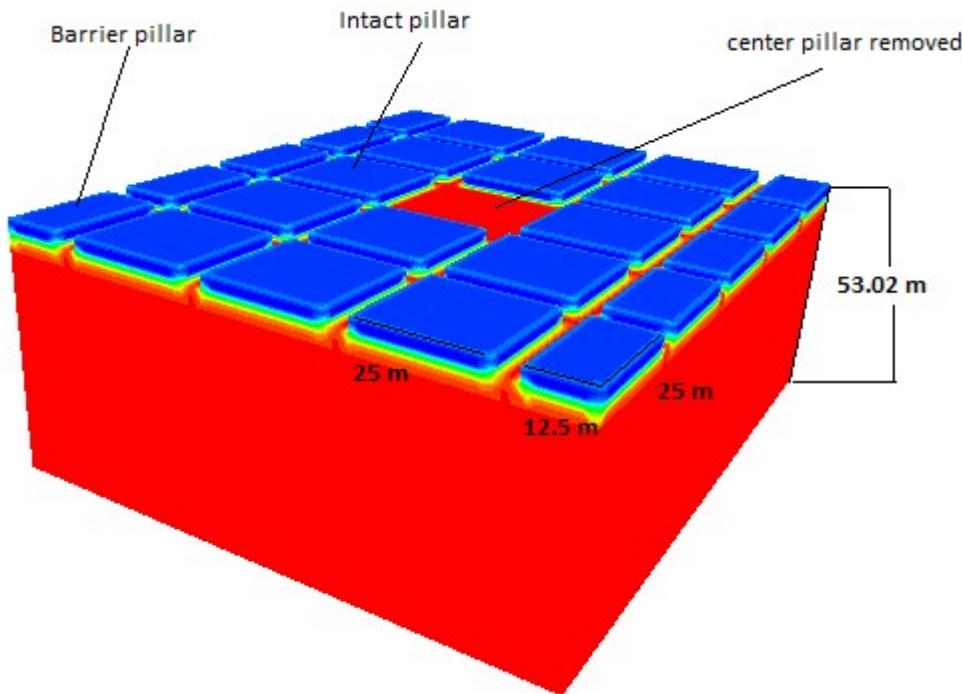


Figure 3.6: The numerical model using FLAC^{3D} for the developed seam where the middle pillar is removed

Using the properties, numerical models were simulated in FLAC^{3D} with different widths and height ratios ranging from 2 to 5. The models were run till they reached.

A fish file is used to determine the average stress and average strain and plotted using the history function in the FLAC^{3D} [19].

The results were imported to excel and the graphs of different width to height ratios of post-failure characteristics are plotted. The steepest part of the post-failure characteristic is the post-failure stiffness [21].

3.6 Parameters Considered

The parameters considered in this project are shown in table 3.4 below where at each respective depth, RMR of 35,45,55 and 65 is considered with a change in young's modulus of 2,3,4 with every RMR.

The stress and convergence of the panel and each pillar are determined by changing the depth, RMR, and Young's modulus respectively.

Table 3.4: Parameters considered

Depth (m)	Pillar size (m)	Rock Mass Rating (RMR)	Young's modulus (GPa)
300	25	35, 45, 55, 65	2, 3, 4
	35	35, 45, 55, 65	2, 3, 4
	45	35, 45, 55, 65	2, 3, 4
600	25	35, 45, 55, 65	2, 3, 4
	35	35, 45, 55, 65	2, 3, 4
	45	35, 45, 55, 65	2, 3, 4
900	25	35, 45, 55, 65	2, 3, 4
	35	35, 45, 55, 65	2, 3, 4
	45	35, 45, 55, 65	2, 3, 4

4. RESULT AND ANALYSIS

The local mine stiffness for the abovementioned conditions are determined and the results are tabulated in Table 4.1. The local mine stiffness is plotted against the post-failure characteristics as shown in Figure.4.1 and 4.2.

Table 4.1: Tabulation of local mine stiffness and post-failure stiffness for a panel at depth 300, RMR 35 and different pillar size and youngs modulus

DEPTH	RMR	Young's Modulus	Pillar Size	Stress	Average convergence with pillar (mm)	Average convergence without pillar (mm)	Local mine stiffness (MPa)	Post failure stiffness
300	35	2	25	10.257606	1.17E-02	8.08E+01	79.39153	383.6199
300	35	2	35	9.4213163	6.31E+00	8.51E+01	146.4281	360.99
300	35	2	45	8.9448066	3.41E+00	9.57E+01	196.194	292.5954
300	35	2	55	8.6420455	2.43E+00	1.08E+02	246.7774	246.3695
300	35	3	25	10.267504	9.20E+00	7.73E+01	94.17425	455.05
300	35	3	35	9.4217511	4.79E+00	8.23E+01	148.9832	367.2891
300	35	3	45	8.9425984	2.43E+00	9.30E+01	199.8334	298.0231
300	35	3	55	8.6407329	1.77E+00	1.06E+02	251.9046	251.4882
300	35	4	25	10.279335	8.11E+00	7.60E+01	94.60407	457.1269
300	35	4	35	9.4143442	4.09E+00	8.10E+01	149.9438	369.6573
300	35	4	45	8.941716	1.99E+00	9.15E+01	202.3993	301.8498
300	35	4	55	8.6422621	1.45E+00	1.04E+02	254.5354	254.1147

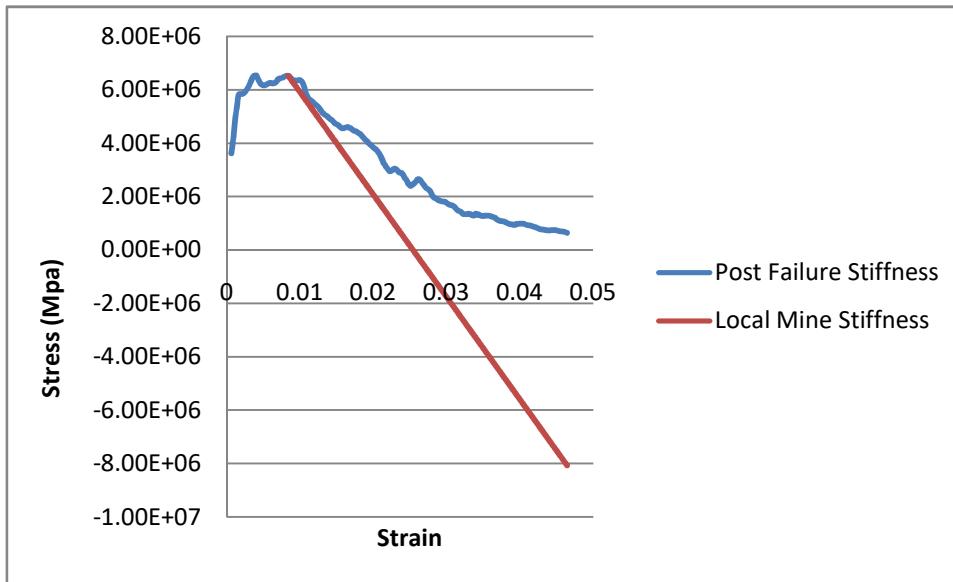


Figure 4.1: Stable or unstable failure for $w/h = 2$ with pillar size $25*25$ at depth 300

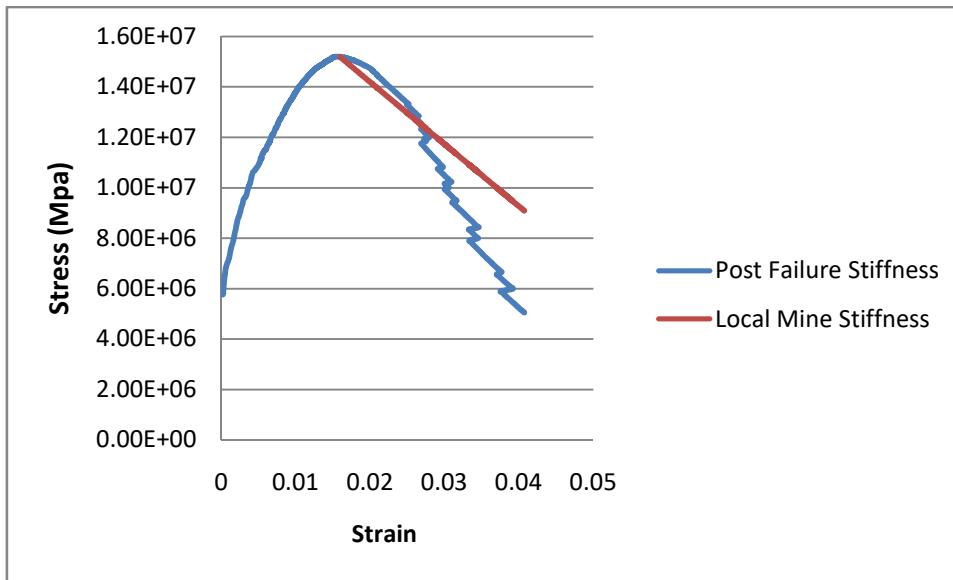


Figure 4.2: Stable or unstable failure for $w/h = 5$ with pillar size $55*55$ at depth 300

4.1 Pillar Size Wise Analysis

For pillar of size 25*25, comparing the w/h ratio from 2 to 5, the local mine stiffness is lesser than the post-failure stiffness in all the graphs in Figure 4.3 (a to g). Hence, it can be stated, the failure in this condition would be stable (i.e., not abrupt and violent) and there is no chance of sudden pillar failure at pillar size of 25*25 at depth of 300 as per the geo-mining parameters used in the model.

For pillar size 55*55 at depth 300, the local mine stiffness obtained from the numerical modelling is almost near to the post-failure stiffness as shown in Figure 4.4(a to g). It can be stated that there is a chance of violent failure of a pillar in the mine. The study is done for pillar of RMR value 25 and young's modulus value 2.

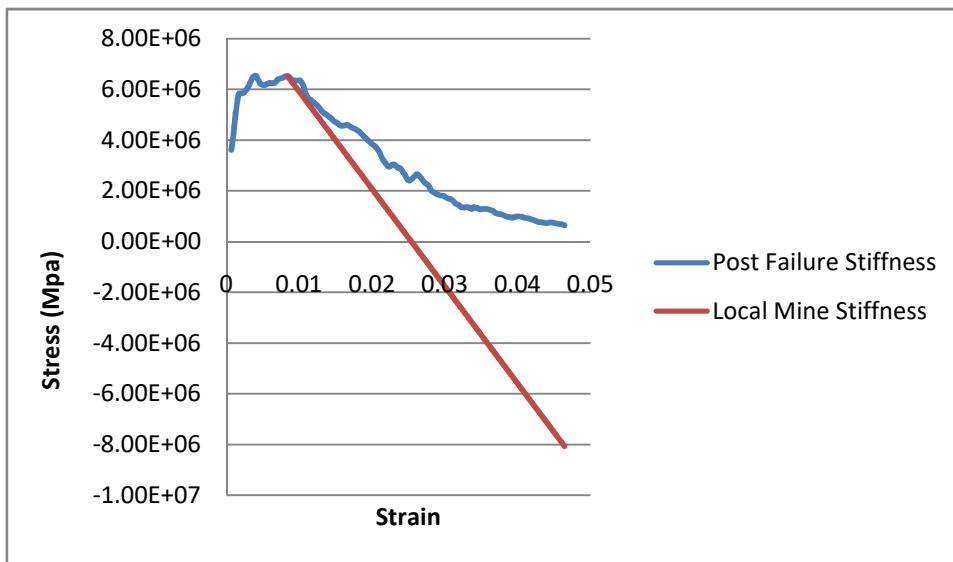


Figure 4.3(a): Stable or unstable failure for w/h = 2 with pillar size 25*25

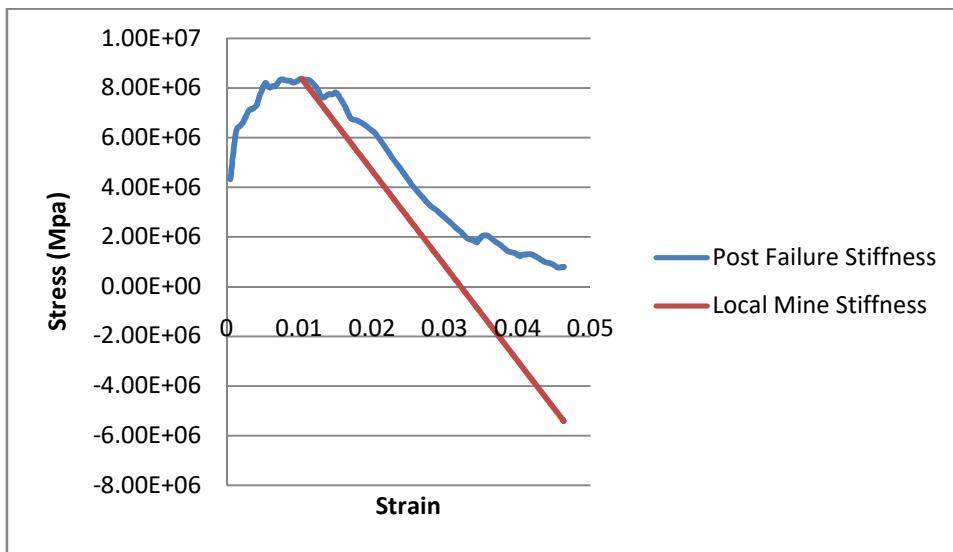


Figure 4.3(b): Stable or unstable failure for $w/h = 2.5$ with pillar size 25*25

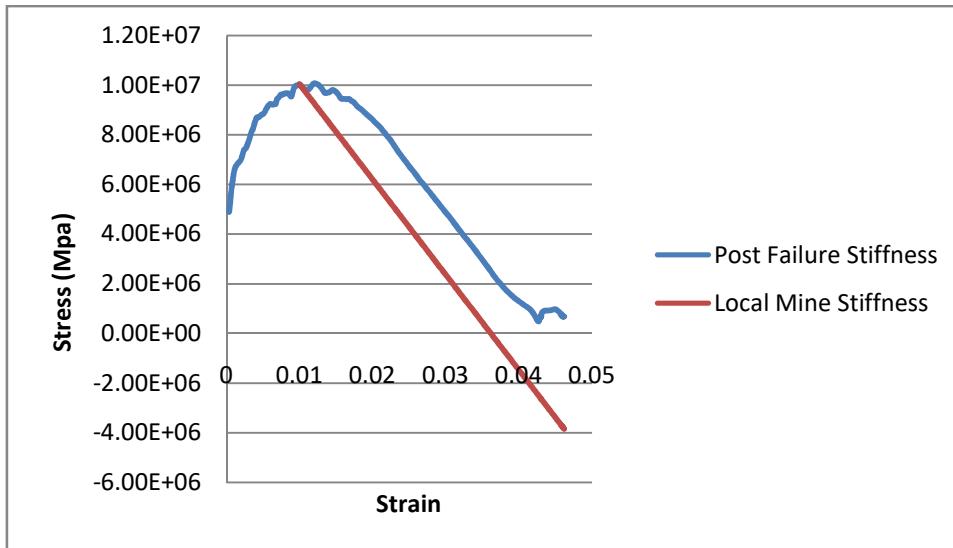


Figure 4.3(c): Stable or unstable failure for $w/h = 3$ with pillar size 25*25

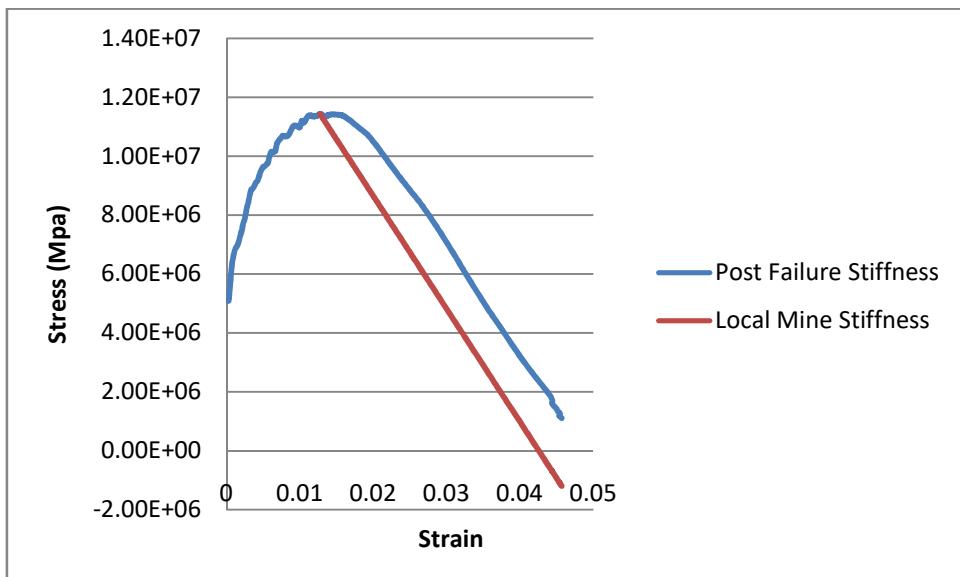


Figure 4.3(d): Stable or unstable failure for $w/h = 3.5$ with pillar size $25*25$

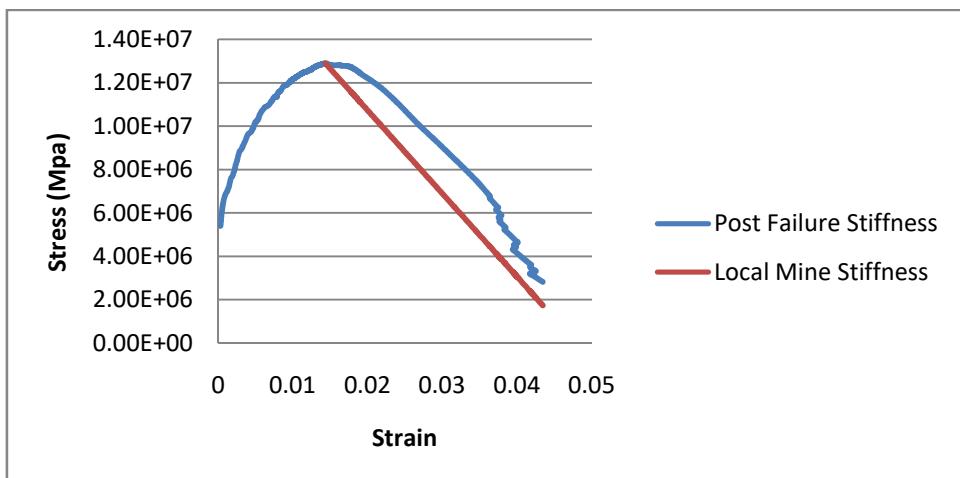


Figure 4.3(e): Stable or unstable failure for $w/h = 4$ with pillar size $25*25$

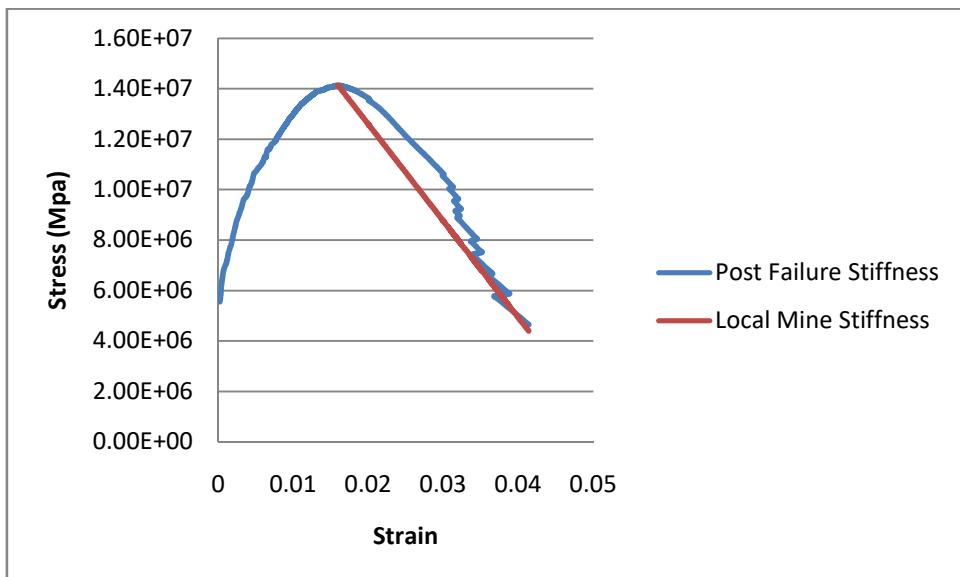


Figure 4.3(f): Stable or unstable failure for $w/h = 4.5$ with pillar size $25*25$

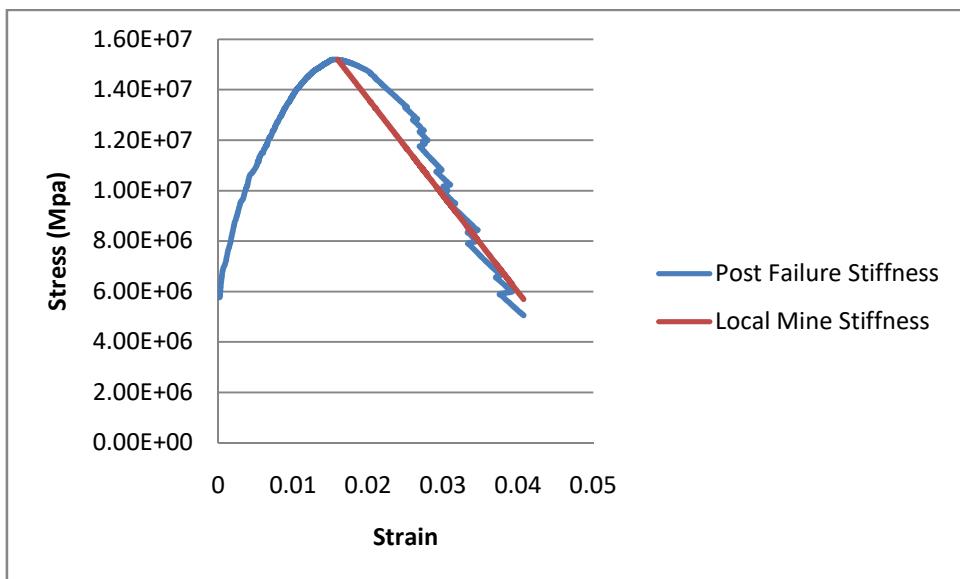


Figure 4.3(g): Stable or unstable failure for $w/h = 5$ with pillar size $25*25$

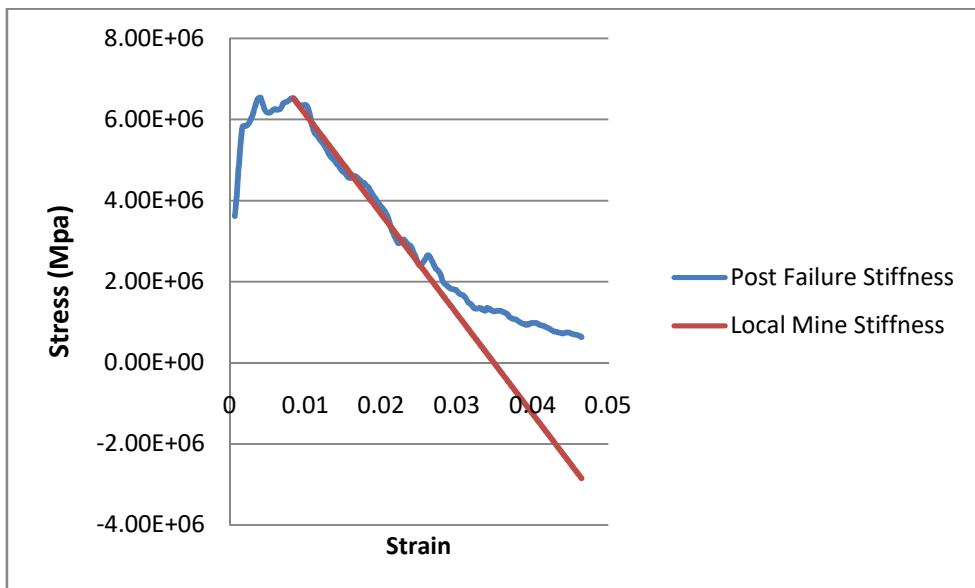


Figure 4.4(a): Stable or unstable failure for $w/h = 2$ with pillar size $55*55$

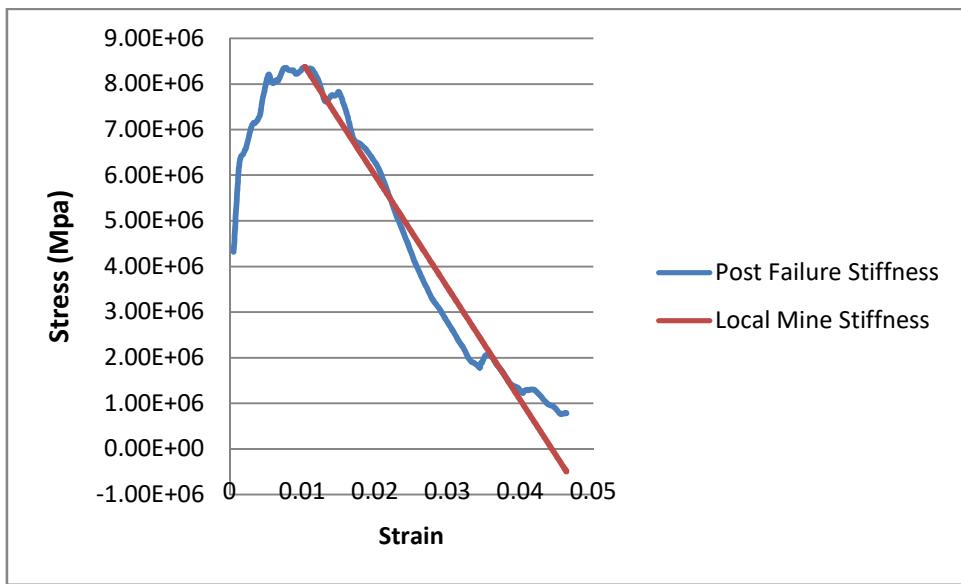


Figure 4.4(b): Stable or unstable failure for $w/h = 2.5$ with pillar size $55*55$

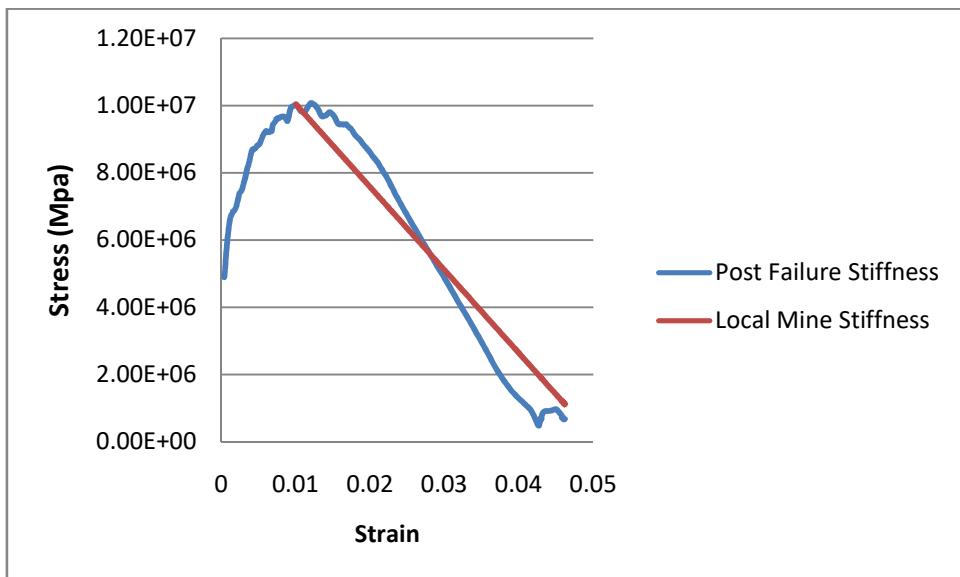


Figure 4.4(c): Stable or unstable failure for $w/h = 3$ with pillar size 55*55

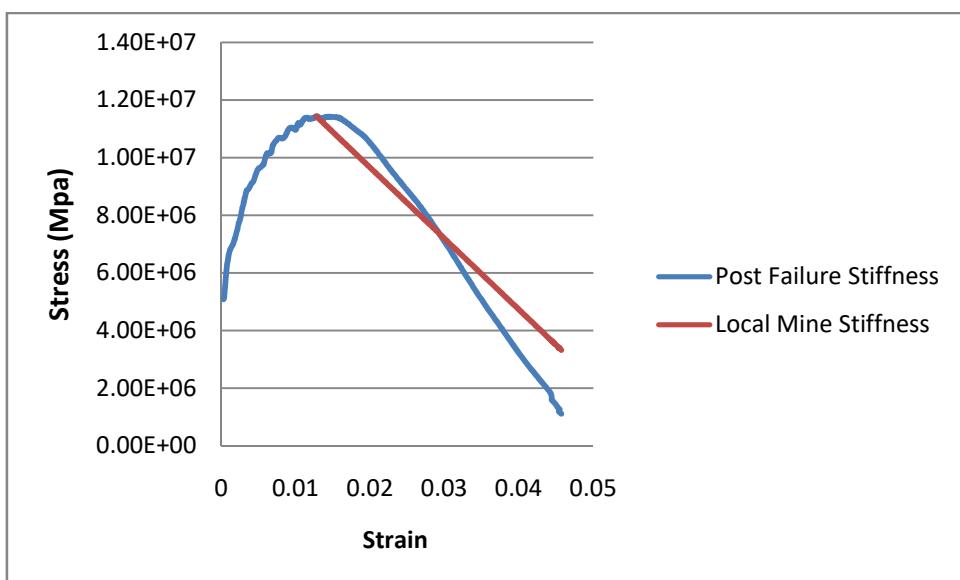


Figure 4.4(d): Stable or unstable failure for $w/h = 3.5$ with pillar size 55*55

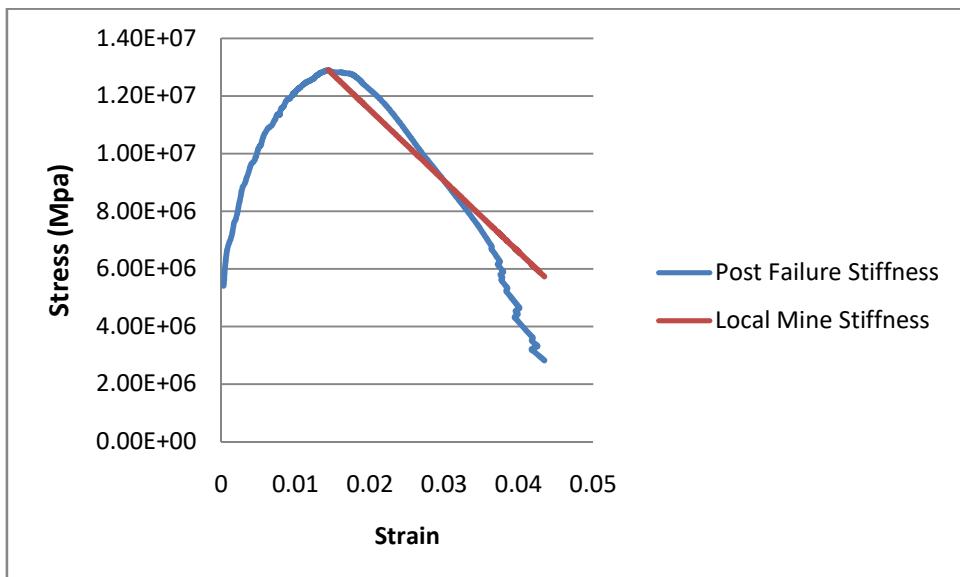


Figure 4.4(e): Stable or unstable failure for $w/h = 4$ with pillar size 55*55

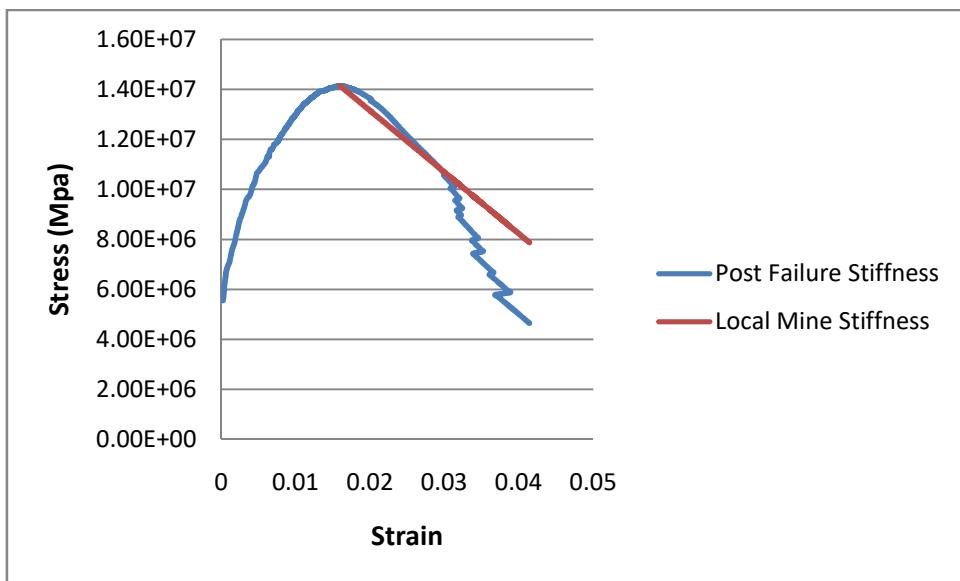


Figure 4.4(f): Stable or unstable failure for $w/h = 4.5$ with pillar size 55*55

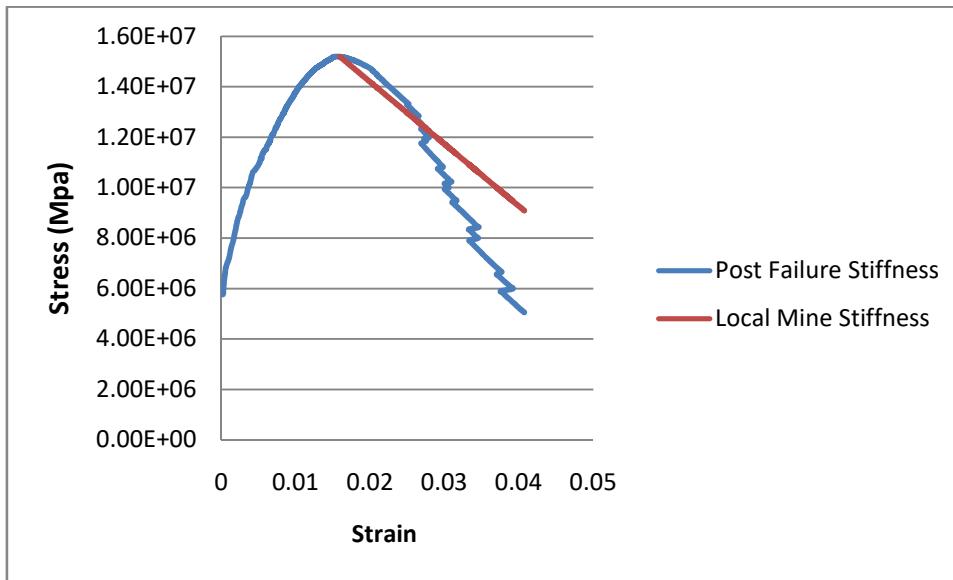


Figure 4.4(g): Stable or unstable failure for $w/h = 5$ with pillar size 55*55

The plotting for local mine stiffness for different pillars and post-failure stiffness is done. Figure shows the stress vs strain graph, the depth is taken as 300m, RMR 35 and Young's modulus of 2. For pillar of size 25*25, stable failure of the pillar happens. For pillar of size 55*55, unstable violent failure will occur. From the study, we can conclude that, as pillar size increases proneness for unstable violent failure increases.

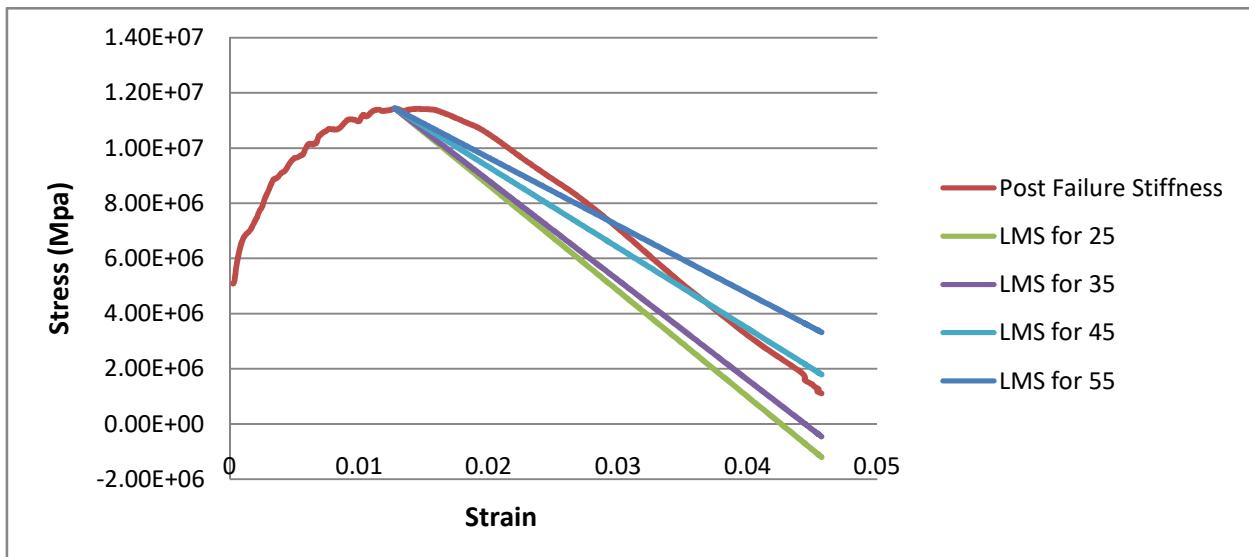


Figure 4.5: Stable or unstable failure for pillar size 25,35,45,55 at depth 300,RMR 35 and youngs modulus 2

4.2 Youngs Modulus Wise Analysis

Plotting of local mine stiffness and post-failure stiffness is done for the pillar of size 25, w/h ratio 5 and at different youngs modulus is shown in figure.12 (a to c). From the study, unstable failure can happen in the case of youngs modulus 2 and stable failure can happen in the case of youngs modulus 4. From the study, it can be concluded that, when youngs modulus increases proneness for stable non-violent failure increases.

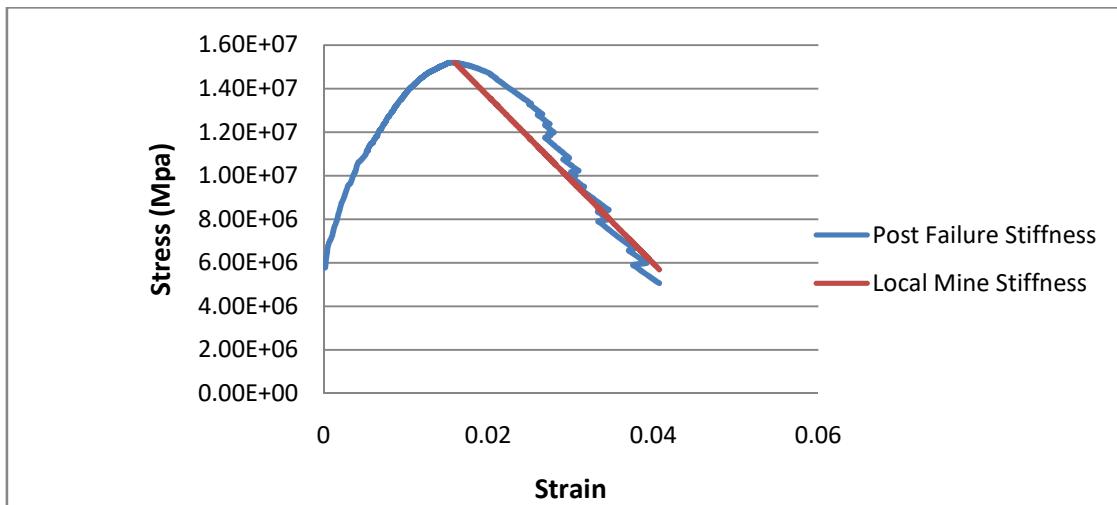


Figure.4.6 (a): Stable or unstable failure for w/h = 5 with youngs modulus 2 at depth 300 and RMR 35

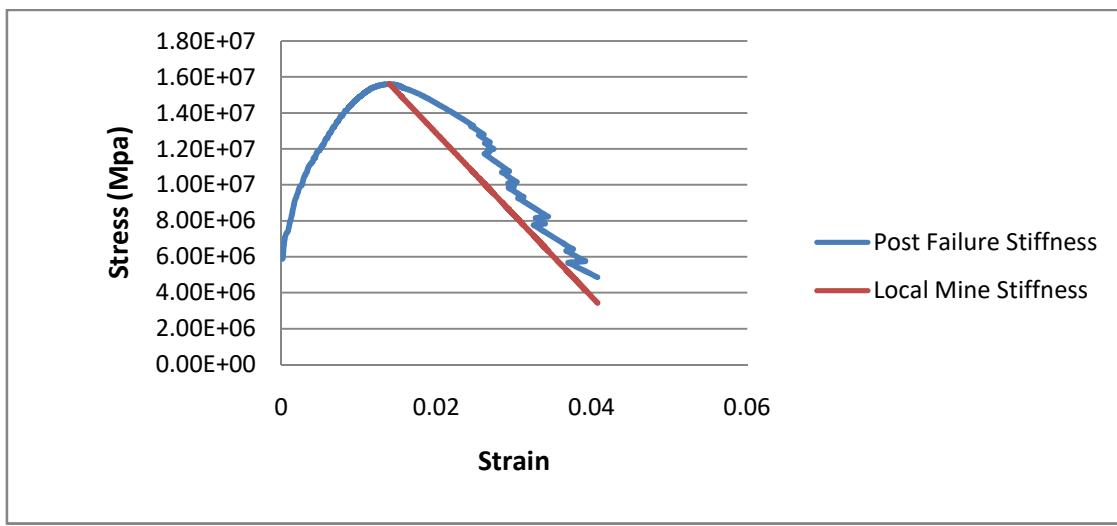


Figure.4.6 (b): Stable or unstable failure for w/h = 5 with youngs modulus 3 at depth 300 and RMR 35

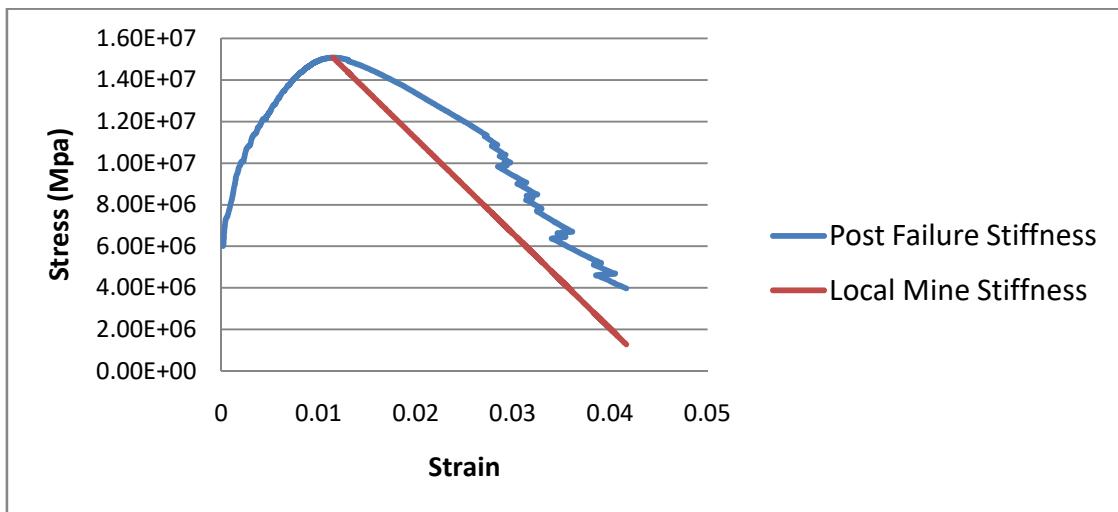


Figure.4.6 (c): Stable or unstable failure for $w/h = 5$ with young's modulus 4 at depth 300 and RMR 35

5.CONCLUSION

The focus of this research was to use numerical modelling to improve the effectiveness and standard safety of dealing with the bord and pillar methods of mining. The pillar stability was evaluated using the details of the digwadith colliery case analysis. For the above-mentioned case study, models were constructed utilising different characteristics and qualities of the coal in the FLAC 3D software. According to the studies, when violent pillar failure occurs during the development or depillaring stages occur at a rapid rate. If the panel's stability is determined before the development stage, a suggestion for a viable and effective support system can be provided in advance. Perhaps if the coal is extracted in this manner, accidents can be prevented and improve output, productivity, and also increasing the safety of the workers. Differentiating parameters such as width to height ratio, RMR, youngs modulus and pillar size are used in this study. Local mine stiffness and post failure stiffness for different conditions are obtained from the study. The conventional way of evaluating the post-failure characteristics is a time-consuming, costly, and ineffective procedure. Now it can be easily determined by the advancement of numerical modelling techniques. When the local mine stiffness is equal or lesser than the post failure stiffness, the pillar fails in an unstable manner. Which is determined and compared using FLAC3D. Due to non-available data such as physio-mechanical characteristics, institute stresses, and so on, the outcome may vary slightly. However, if all of the essential data is available, numerical modelling can be used to reasonably predict the stability of a panel in advance.

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